Building a Model of the Lunar Core Using Laser Ranging Observations of the Moon

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Abstract—We discuss the problem of developing a theory of the Moon's physical libration for the model that takes into account the presence of a liquid core. Facts are presented that testify to the existence of a small lunar liquid core. The possibilities of determining from observations the effects of the existence of the lunar core and determining its characteristics, primarily the flattening of the core, are considered. A description of the solution to the problem of including a two-layer model for the task of numerical solution of the equations of physical libration of the Moon is given.

In addition to obtaining high-precision information about the harmonics of the selenium potential [1], many important facts have been obtained about the structure of the lunar body. In the course of long-term lunar laser ranging (LLR), and the determination of the lunar physical libration (LPL) on the basis of these data, signs of dissipation of the lunar rotation were revealed, the cause of which is not only tidal friction, but also, presumably, turbulent processes at the boundary of the solid mantle and a hypothetical (at that time) liquid core. By means of computer simulations, JPL NASA specialists were able to estimate the size of the outer liquid and inner solid cores of the Moon, and their chemical composition. Direct evidence of the Moon's core was obtained after reprocessing the seismograms of the "Apollo" mission with the help of new methods for processing the reflected and converted seismic energy [2]. It turned out that the Moon has a hot metallic core, which is surrounded on the outside by a partially molten shell (10−30%) and contains a solid iron core inside, and it was also possible to estimate the size of these three components of the core—330 ± 20, 480 ± 20, 240 ± 10 km, respectively. Confirmation of these data was obtained by Chinese scientists [3], who, based on a high-resolution GL1500E gravitational field model (GRAIL mission) considering the data of the LLR, estimated in the same values not only the sizes of the three components of the lunar core, but also the density and chemical composition of each of them. Kronrod et al. [4]. On the basis of various methods of computer simulation of the physicochemical and thermal state of the lunar

core, they estimated the range of possible size of the inner core and the chemical composition, and showed that the size of the core, which is less than 1% of the mass of the Moon, is weakly dependent on both the chemical composition and the thermal regime of the mantle. Despite these studies, the following questions remain unsolved—what is the compression of the core, its rotation speed, the inclination of the axis of rotation to the axis of rotation of the mantle, etc.

The core of the Moon can also manifest itself in the form of additional modes of free libration, which arise due to the mismatch of the axes of rotation of the outer and inner cores with the rotation of the mantle (see figure). Estimates of the periods of such librations can be made theoretically [5]. Estimates of the periods of these librations have been made, the most significant of which is the so-called Free Core Nutation (FCN). Its period in the inertial coordinate system is estimated by the formula:

$$P_{\text{FCN}} = T_\Omega(1 + R_c)/f_c. \tag{1}$$

Simulations (see Fig. 1) have shown that, due to the relatively slow proper rotation of the Moon $T = 27.32$ days, the FCN period with a slight compression of the core $f_c = 3.8 \times 10^{-4}$ reaches hundreds of years and is weakly dependent on the size of the core R_c. Therefore, it is quite difficult to detect this harmonic from observations. Analysis of the GRAIL data allowed us to estimate the value of $P_{\text{FCN}} \sim 300$ yr.

In order to achieve the necessary accuracy, it is necessary to consider all the harmonics associated with the nucleus. This approach based on Barkin's
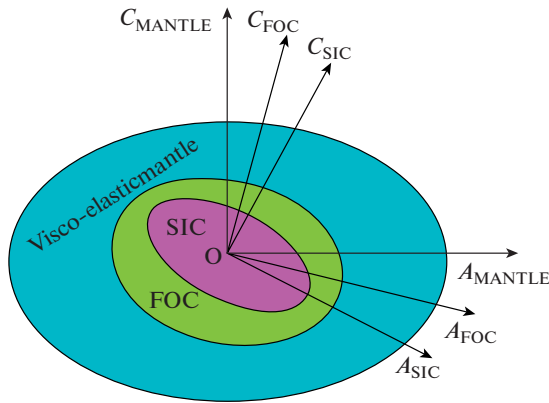
Fig. 1. Cross-section of the Moon along its axis of rotation. The fluid outer core (FOC) and the solid inner core (SIC) are axisymmetric: $A_{\text{MANTLE}} = B_{\text{MANTLE}}$, $A_{\text{FOC}} = B_{\text{FOC}}$, $A_{\text{SIC}} = B_{\text{SIC}}$—are the equatorial moments of inertia for the mantle and both nuclei. The moments C_{MANTLE}, C_{FOC}, C_{SIC}—are the maximum moments of inertia.

theory is described in detail in [6]. Theoretical data obtained by comparing the results of the LPL analytical theory constructed for a model of the Moon containing a liquid core with semi-empirical series—SES) [7] built on the basis of the LLR data and the dynamical theory of the Moon's spin-orbital motion DE421 [8] were analyzed. SES, in addition to the known members of the forced and free libration of the Moon, contain harmonics with small amplitudes, the nature of which could not be previously determined, they were designated as unknown harmonics by the symbol *Un*. It turned out that most of the unknown harmonics coincide with the harmonics calculated theoretically, due to the presence of a nucleus near the Moon. The amplitude of such harmonics in the SES is indeed small—less than 30 ms, but nevertheless the coincidence of their frequencies with the calculated ones indirectly indicates that free librations of the Moon in rotation exist, as well as free librations in the mantle of the Moon.

Analytical studies related to the inclusion of the two-layer Moon model in libration equations were continued by us as part of the numerical approach. A method was developed for constructing the theory of LPL as part of the model of the solid Moon with an approximate account of its tidal deformation [9]. In the scope of the Hamiltonian approach, it was necessary to improve the LPL model by including a liquid nucleus in it. To do this, we used the Poincare model. This model describes the free rotation of a liquid core in the solid mantle body. In the scope of this model, we were able to build 12 equations for the six parameters of the Moon's rotation (three parameters each for the solid mantle and the liquid core and their corresponding six conjugate pulses). However, in order to

solve these equations numerically, we needed to obtain the initial values of the parameters of the rotation of the nucleus, which have not yet been determined from observations. Inasmuch as the nucleus is very small, its interaction with external bodies is usually neglected, and only its free rotation is considered. To estimate the compression of the kernel in (1), the data of the theory [5] and the value of P_{FCN} from this theory were used. However, it is not possible to derive the amplitude and phase values of FCN from analytical calculations. Therefore, the characteristics of the lunar core and its free libration from the work [6] were used for their calculation: the radius of the core is 240–330 km (calculated from the reprocessing of seismic measurements), compression—7×10^{-4}, for FCN period = 77757.032 days (205.7 yr), amplitude = 0'0395, phase = −134° (epoch 2000). For comparison, the FCN period calculated for the specified kernel parameters according to the model [5] ranges from 144.02 to 200.2 yr.

The second method of determining the initial values for the parameters of the rotation of the nucleus was based directly on the use of semi-empirical series [7]. The series are an analytical description of the LPL for its three parameters—longitudinal libration (τ), latitudinal libration (ρ), and libration at the node (σ). Following the conclusions [6] that those members of the SES that are designated as members of unknown nature (*Un*) are derived from free libration, the initial values of the libration parameters for the J2000 epoch were calculated. The amplitudes of the Un members do not exceed tens of milliseconds, therefore the final values of the initial parameters are at the level of a millisecond or less: $\tau = 0.76$, $\rho = 0.12$, $\sigma = 3.41$ ms. According to the theory [6], the same values were equal to 1.05, 0.23, and 7.01 ms, respectively.

In the simulation process the parameters of the lunar core and comparing them with the results of calculations using semi-empirical series, the conditions under which the minimum in residual differences is achieved for frequencies defined as harmonics of unknown nature are determined. According to the conclusions of the analytic theory [6], it is these harmonics that confirm the presence of a liquid nucleus. In simulation, the optimal values for the radius and compression of the nucleus were obtained and the values of the period of free nutation of the nucleus were specified. These data increased the probability of detecting kernel parameters in PhLL observations. Therefore, as a result of the study, we obtained information about the possibility of detecting harmonics in PhLL observations that confirm the presence of a lunar core. With the help of simulation of the nucleus parameters, the dynamics of such harmonics was investigated and the prospects of planned experiments using lunar telescopes to find the parameters of the free nutation of the nucleus and refine its characteristics were determined.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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