DEVELOPMENT OF A HIGHLY ACCURATE THEORY OF THE PHYSICAL LIBRATION OF THE MOON FOR ITS USE IN THE SYSTEM OF COORDINATE AND TIME SUPPORT

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Introduction: Modern experimental studies of the external and internal characteristics of the Moon are connected with plans for its exploration in the near future. At the beginning of the new millennium, a series of space programs was launched aimed at the global study of the Moon. Lunar navigation at observations from the lunar surface requires a high accuracy of the theory of spin-orbital motion, including the physical libration of the Moon (PLM) [1, 2]. This, in turn, requires the improvement of the numerical and analytical theories of the rotation of the two/three-layered Moon, which will allow them to be used in the processing and analysis of high-precision observations [3], obtained in the course of long-term lunar laser ranging, as well as those observations that will be produced in the planned new experiments, such as the SELENE-II project. The modeling performed in this work will contribute to establishing the optimal conditions for placing measuring equipment on the lunar surface and developing an efficient schedule for upcoming observations [4, 5].

Methods: When developing theories of spin-orbital evolution and tidal dissipation of the multilayer Moon, constructing the corresponding analytical and numerical theories of the rotation of the Moon, and subsequently other multilayer bodies of the Solar System, the effects of tidal dissipation and the influence of processes at the core-mantle boundary on the rotation of a celestial body must be taken into account [6, 7]. To obtain an analytical solution in the form of a Poisson series, methods for the solid state of the Moon were used [8]. When describing the orbital motion, the analytical theory of the motion of the Moon was used [9]. Analytical forms of the main characteristics of the orbital resonance of the Moon were obtained, which are calculated from the observational data of SELENE, LRO, GRAIL space missions, and compared with the dynamic ephemeris DE30/431 (JPL NASA). To include weaker, but more difficult to describe effects, a numerical theory of physical libration was developed based on new information technologies and high-level programming languages [10].

Results: The main algorithms for constructing the PLM numerical theory for the deformable Moon with the inclusion of direct and indirect planetary perturbations were developed. The corresponding analytical approach was also developed in the same direction [11]. The analysis of the parameters of deviation from the hydrostatic equilibrium figure of the Moon and the development of methods for taking into account the viscoelastic properties of the lunar body to include them in the theory of physical libration were carried out. As a result, numerical and analytical theories of the elastic Moon were constructed. Computer simulation of PLM using space astrometry methods will make it possible to determine the influence of the lunar quality factor Q, the elasticity coefficient (k2 and h2 Love numbers), and the effects of viscosity (in particular, the retardation angle). These effects describe the deformation of the body of the rotating Moon and define its response to various external and internal perturbing factors.

Conclusions: As a result, these data will make it possible to determine most effectively the location and parameters of the measuring equipment as well as to set up a schedule of observations for the planned experiments. Inclusion of two-layer structure – viscoelastic mantle and liquid core – into the model of the Moon will require the development of a method for taking into account the description of dissipative processes in the theory of the rotation of the Moon, caused by both the viscosity of its mantle and processes at the “core-mantle” boundary. In the future, the effects of the influence of the core on the lunar rotation will be revealed and estimates of the parameters of the core will be produced – its size, state of aggregation, and chemical composition. An important outcome of the development of the PLM theory will be the determination of the parameters of the free core nutation and the search for the possibility of detecting free nutations from the planned space observations [12].

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PHYSICAL LIBRATION OF THE MOON TAKING INTO ACCOUNT THE EFFECT OF EARTH COMPRESSION

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Introduction: One of the important and complex tasks of celestial mechanics is the description of the translational-rotational motion of the Moon in the gravitational field of the Earth and the Sun, taking into account their size and shape. At the same time, significant deviation of the shape of the Moon from spherical symmetry is considered as well [1]. The solution to this problem is basically determined by the quality, accuracy, and quantity of the observed data. The observations allow controlling the accuracy of the numerical theories of physical libration of the Moon (PLM) applied. A large number of lunar missions have been completed recently (in particular, in 2019 the Chang'E-4 mission was carried out) [2]. However, it has not yet been possible to completely solve all the fundamental issues related to the construction of the PLM numerical theory.

Methods: To solve the problem of creating a theory of the physical libration of the Moon, taking into account the influence of the Earth's compression, we created a theory of the Moon's rotation within the framework of an absolutely rigid body based on the analytical theory of the Moon's motion [3]. In this case, it was necessary to take into account in the first approximation the elastic properties of the Moon and perturbations from the planets by switching to modern orbital theories of motion DE (dynamic ephemeris) developed at the NASA Jet Propulsion Laboratory, as well as to the EPM numerical theory [4] developed at the IAA RAS. It should be noted that the use of the point interaction model in the calculation of gravitational forces does not work in all cases. If celestial objects are seen relative to each other with a significant angular radius, then the effects of the deflection of a spherically symmetric body will be quite significant. If these effects can be neglected, then the gravitational potential of the main body is considered as a harmonic expansion in terms of spherical functions, and the other body is taken as a point body. In this paper, the differences between the Earth and a point body were considered.

Results: As a result, it was found that physical libration in latitude contributes about 0.15 arcseconds over an interval of about 10 years, while the contribution to physical libration in longitude is an order of magnitude smaller. The calculations were made to take into account the precession-nutation movement of the Earth's pole. Estimates of this phenomenon showed that this contribution to PLM parameters is much smaller. The obtained parameters of the physical libration of the Moon, taking into account the Earth's compression, were compared with the dynamic ephemeris DE421 [5] and it turned out that the contribution from the Earth's compression is much less than the accuracy of the very PLM theory. However, taking into account the compression of the Earth is important at a qualitative level, yet quantitatively this effect does not have a very significant impact. At the same time, possible errors that lead to high residual differences are mainly associated with the presence of inaccurate initial data. We showed that if one takes into account the resonance contribution to PLM from the initial data, then the amplitude of the residual differences decreases down to 0.13 arc seconds in longitude, though over a short time interval. Further, the error increases. All the results obtained are new and can be used in the study of selenophysical parameters, including investigations of the lunar core structure [6].

Conclusions: In this paper, we simulated the effect of the Earth's compression on the physical libration of the Moon [7]. It is shown that the contribution of the effect is approximately two orders of magnitude smaller than the current residual difference with the modern DE421 theory [8, 9]. All the necessary formulas describing this effect are derived. Due to the fact that the PLM equations are described in the Hamiltonian formalism, and foreign scientists mainly use classical equations, which include moments from the perturbing body, we also verified that the formula we obtained is not contradictory [10].

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