ISSN 1063-7729, Astronomy Reports, 2020, Vol. 64, No. 12, pp. 1078–1086. © Pleiades Publishing, Ltd., 2020.
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Lunar-Based Measurements of the Moon's Physical Libration: Methods and Accuracy Estimates

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Received December 19, 2019; revised July 7, 2020; accepted July 30, 2020

Abstract—Computer simulation results are reported for planned lunar-based observations to be conducted using an automated zenith telescope that can be placed at any lunar latitude. Benefits of lunar-based observations are shown, in comparison with lunar laser ranging. It is investigated whether, and how effectively, these observations can be used to determine the lunar rotation parameters (physical libration). The accuracy requirements for these observations are analyzed in view of the accuracy requirements for determining the lunar rotation parameters. The necessary number of telescopes, as well as their optimal locations, is assessed.

DOI: 10.1134/S1063772920120094

1. INTRODUCTION

Lunar-based measurements of physical libration imply that measuring instruments are placed onto the lunar surface. Observations from the lunar surface are free from atmospheric fluctuations and need not accommodate the Earth's orbital and rotational motion.

The new-type observations offer yet another advantage—they are independent of lunar laser ranging (LLR) and can provide even higher accuracy. It is important that we consider here these two matters of principle. Firstly, lunar-based observations are independent of LLR and, hence, make it possible to clarify whether LLR data have systematic errors. Such errors may arise because

(1) LLR is not sensitive to the direction perpendicular to the Earth–Moon line; consequently, observations are less sensitive to librations around the Earth–Moon line.

(2) It is difficult to perform LLR during new moon and full moon; therefore, observations are synchronized with the libration period.

(3) LLR amplitudes depend on uncertainties in the positioning of lunar-based reflectors.

Secondly, lunar-based observations enable the separation of different parameters and different frequencies. Coupled with an increase in the quality of LLR, this separation can significantly improve the quality of observations over the Moon's rotation.

Space agencies and scientific organizations in many countries are examining the various types of such measurements.

Thus, in 2018, NASA announced the LSITP (Lunar Surface Instrument and Technology Payloads) project. This project plans to deliver landing modules onto the lunar surface with equipment for completing not only scientific but also commercial tasks. The LSITP is scheduled for implementation in the early 2020s.

Following the success of the *SELENE* (*Kaguya*) mission in 2008–2010, Japanese researchers considered the possibility of using a low-orbit satellite and lunar-surface radio beacons to implement the inverse VLBI method. However, computer simulation of this experiment [1] showed its low efficiency for determining the lunar rotation parameters.

However, another project planned by the Japanese space agency (JAXA)—the ILOM (in situ Lunar Orientation Measurement)—indeed showed good prospects in this area. Within the ILOM project, it was proposed to put a zenith telescope on one of the lunar poles [2]. It was planned to measure, within that project, the selenographic coordinates of stars with a high accuracy and use these data to determine the lunar rotation parameters. There are many benefits to positioning an automated zenith telescope (AZT) on a pole.

1. Here, good conditions exist for placing measurement instruments due to the existence of both permanently shadowed and permanently illuminated zones.

2. Here, stars move slowly, and the number of stars is limited by the pole's precession ring, beyond which the Moon's diurnal rotation does not take them. All these factors contribute to efficient detection and accurate identification of stars.

3. The error in obtaining the lunar rotation parameters when processing a single measurement is effectively neutralized by statistics on large stellar samples, thus reducing the resulting error to almost zero [3].

In [3–6], the possibilities of the ILOM project were investigated by computer simulation. It was shown that, apart from suitable conditions for technology positioning of the instruments on the poles, this project offers good chances for determining the lunar rotation parameters in terms of latitude and tilt. Nevertheless, the described experiment has one crucial setback—as shown by calculations, polar observations cannot be used to determine the third parameter of the Moon's rotation, i.e., longitudinal libration, which carries in itself a lot of useful information about the lunar structure.

In this regard, it seems interesting to explore the possibilities of an experiment in positioning one or several AZTs at other lunar latitudes. This paper investigates the quality aspects of determinations of the lunar rotation parameters in the case of nonpolar telescope positioning and discusses the search for an optimal telescope localization on the Moon.

2. CONSTRUCTING A MATHEMATICAL MODEL OF THE EXPERIMENT

The lunar exploration history knows no cases of applying AZTs to determine the lunar rotation parameters; therefore, computer simulation techniques present one of the most effective ways to study the fundamental possibility, as well as efficiency, of using AZTs for these purposes. Simulation of planned observations serves to:

(1) Determine the necessary number of AZTs and their optimal positioning.

(2) Develop a program of observations and determine their duration.

(3) Substantiate the accuracy requirements for the observations in view of the accuracy requirements for the lunar rotation parameters.

Solving the formulated problems depends, firstly, on correct construction of the transition matrix between the lunar and inertial coordinate systems and, secondly, on correct determination of the telescope coordinates in the lunar system.

The technical characteristics of the telescope in the simulation are the same as in the ILOM experiment [7]: the field of view is 1°; the accuracy of one measurement in the field of view of the CCD matrix is no worse than 10″; the telescope has an azimuthal mounting, with the tube being directed to the zenith of the observation site. The duration of the observations depends on the quality of the equipment that maintains the operation of the measuring instruments. The desirable duration varies from a year to a year and a half, so as to be able to refine the long-period components of the lunar rotation parameters (LRPs) from the observations.

The task of enabling the analysis of effects attributable to features of the internal structure of the lunar body necessitates an LRP determination accuracy of no less than 1″.

We now define some of the fundamental positions for the localization parameters of the lunar-based telescope. The position of the telescope in a dynamic coordinate system (DCS) formed by the Moon's principal axes of inertia is defined by the longitude l_T and latitude b_T (Fig. 1).

The position of the DCS relative to, in our case, the ecliptic coordinate system is defined by the Euler angles Ψ, Θ, and φ,

$$\begin{aligned} \Psi &= \Omega + H \cdot \sigma, \\ \Theta &= I + \rho, \\ \varphi &= l_T + F + 180^\circ + \tau + H \cdot \sigma, \end{aligned} \quad (1)$$

which include the lunar physical libration (LPL) angles: $\tau(t)$, $\rho(t)$, and $I\sigma(t)$, in longitude, inclination, and node, respectively. It is the libration angles that are the LRPs. I is the mean inclination of the lunar pole P_{lun} to the ecliptic pole Π_{ecl}. In (1), it is assumed that $H = -\frac{1}{I}$. The parameter F is the latitudinal argument, i.e., the angular distance of the Moon's mean longitude from the ascending node of the lunar orbit Ω. The angles $\tau(t)$, $\rho(t)$, and $I\sigma(t)$ for a given time interval are calculated using the analytical theory of physical libration [8, 9].

The field of view of the telescope (FVT) (T) in degrees can be varied, if necessary, in the course of the simulation.

Since the location of the telescope will change in the course of the simulation, it makes no sense to solve the sampling problem for observations of specific stars from stellar catalogs with all the necessary reductions. Therefore, instead of choosing stellar coordinates from catalogs, we simulate the ecliptic coordinates of any given number of stars using a random number generator that simulates the ecliptic longitudes λ and latitudes β of "stars" in a band of width equal to the FVT, along the line of the precessional motion of the telescope (Fig. 2).

The system of selenographic stellar coordinates (x_1, y_1, z_1) in the FVT is defined as follows: the z axis aims to the zenith of the telescope site T and serves as its axis; the x axis goes along the meridian on which T stands; and the y axis forms a right-hand coordinate system (Fig. 1b).

The position of the telescope is specified relative to the DCS by the longitude l_T and latitude b_T. We consider an ideal situation where the coordinates l_T and b_T are known with absolute accuracy. This idealization is

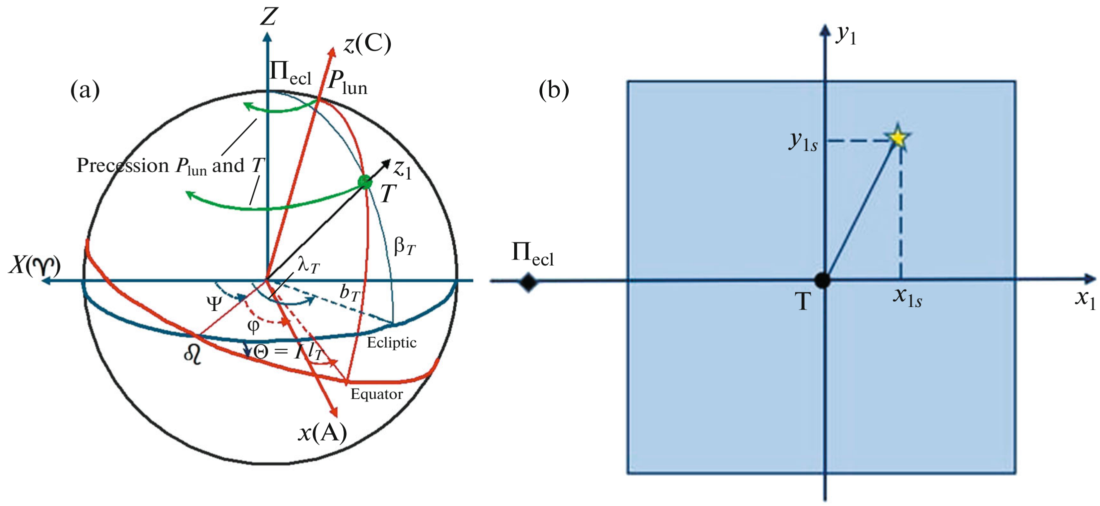

Fig. 1. (a) Position of the telescope T relative to the ecliptic and dynamic coordinate systems. The lines are shown of the lunar pole's retrograde precession and the corresponding motion of T. (b) Mapping of the FVT and stellar selenographic coordinates in the telescope coordinate system.

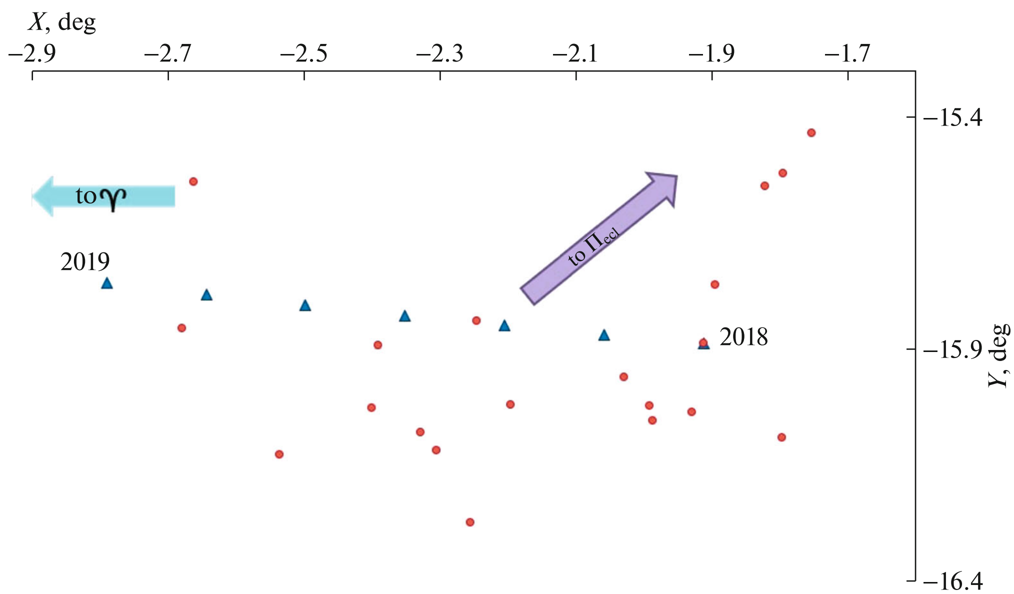

Fig. 2. Motion of the telescope in the ecliptic coordinate system, and the stars whose coordinates were generated in such a way that they fall within the FVT during the designated observation period in 2018–2019.

quite far from reality because the coordinates of objects in the DCS have large errors [10]; thus, such a model can only be regarded as a first approximation.

Based on these assumptions, we can derive the selenographic coordinates of a star in the telescope coordinate system (TCS) from the equation system

$$x_1 = a_1(\cos\lambda\cdot\cos\varphi + \cos\Theta\cdot\sin\lambda\cdot\sin\varphi) - b_1\sin\Theta\cdot\sin\varphi - c_1\sin\Theta\cdot\sin\lambda - d_1\cos\Theta,$$

$$y_1 = -a_2\cos\lambda\cdot\sin\varphi + b_2\cos\Theta\cdot\sin\lambda\cdot\cos\varphi - c_2\sin\Theta\cdot\cos\varphi, \quad (2)$$

$$z_1 = a_3(\cos\lambda\cdot\cos\varphi + \cos\Theta\cdot\sin\lambda\cdot\sin\varphi) - b_3\sin\Theta\cdot\sin\varphi + c_3\sin\Theta\cdot\sin\lambda + d_3\cos\Theta,$$

which was obtained using the rotation matrices in going from the ecliptic coordinate system to the DCS [4] via the Euler angles (1) and then from the DCS to

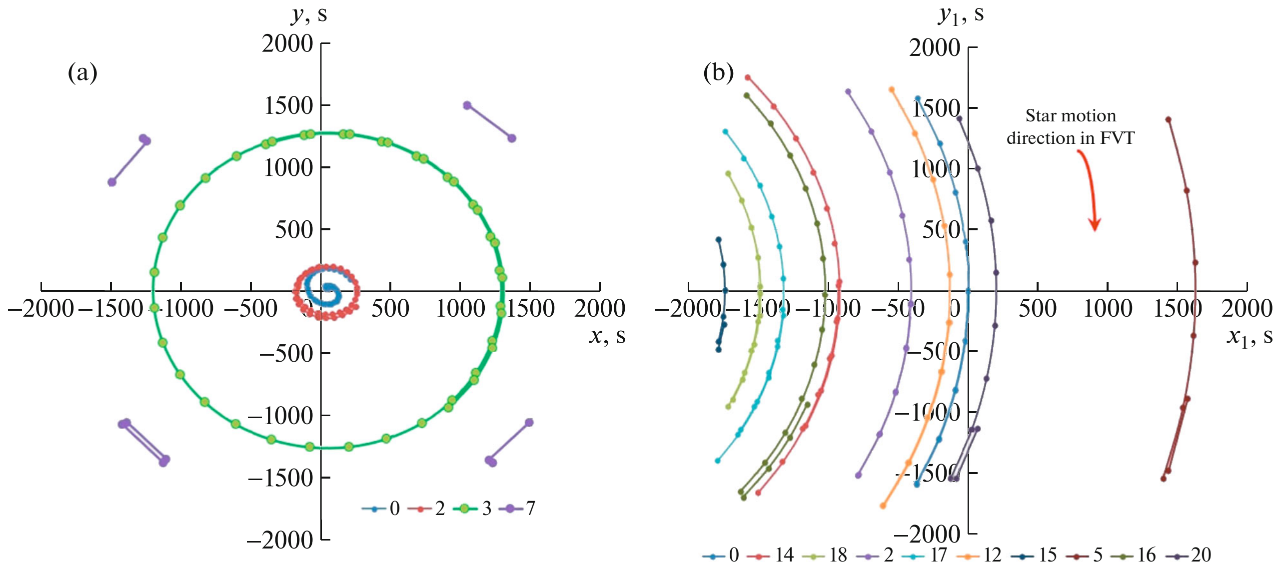

Fig. 3. (a) Tracks of four polar stars observed with a polar telescope ($l_T = 0°$, $b_T = 90°$) for 33 days. Star 7 appears in its field of view only periodically. (b) In the field of view of a telescope with the coordinates $l_T = 0°$, $b_T = 89°$, the tracks change their shape; some of the stars appear twice within the specified period.

the TCS via the angles l_T and b_T. Here we introduce the following notation:

$$\begin{gathered} a_1 = \sin b_T \cdot \cos\beta; \quad b_1 = \sin b_T \cdot \sin\beta; \\ c_1 = \cos b_T \cdot \cos\beta; \\ d_1 = \cos b_T \cdot \sin\beta; \quad a_2 = \cos\beta; \quad b_2 = \cos\beta; \\ c_2 = \sin\beta; \quad d_2 = 0; \quad a_3 = \cos b_T \cdot \cos\beta; \\ b_3 = \cos b_T \cdot \sin\beta; \quad c_3 = \sin b_T \cdot \cos\beta; \\ d_3 = \sin b_T \cdot \sin\beta, \end{gathered} \tag{3}$$

$\lambda = \lambda_0 - \Psi$, λ_0, β are the ecliptic coordinates of the star.

3. ANALYSIS OF STAR TRACKS DEPENDING ON THE LATITUDE OF THE TELESCOPE SITE

The resulting Eqs. (2) and (3) can serve to reproduce the construction of star tracks, which is the *direct problem* of the simulation.

If the telescope stands close to the pole, polar stars, as seen through it, experience a shift equal to the polar distance of the telescope. In the TCS, the visibility of polar stars in its FVT of 1° is shown in Fig. 3.

The tracks are no longer spirals; within one month, a star can cover a part of its path, leave the FVT, and then reappear. When moving to southern latitudes, the pattern of the star tracks undergoes no significant changes; the lines become straighter. Due to the increase in the radial velocity of rotation of the telescope, at low latitudes the instrument is able to detect only 1 to 4 appearances of a star in 2 months, even if records are taken quite frequently, every 15 min (see Fig. 4).

4. SOLVING THE INVERSE LPL PROBLEM FOR NONPOLAR TELESCOPES

At the stage of the *inverse problem,* the rectangular coordinates of the star, which were calculated in the direct problem, are considered as "observed" stellar coordinates and are used to calculate the "unknown" LRPs. In Eq. (2), these "unknowns"—$\tau(t)$, $\rho(t)$, and $I\sigma(t)$—are part of the relations for the Euler angles (1) and can be derived from these equations by the approximation method [5, 6]. In the course of the simulation, it was shown that the system of nonlinear equations (2), despite the close-to-zero Jacobian, allows for a convergent iterative solution by the gradient method [10].

The technology developed by the authors and the corresponding software for realizing the gradient method was applied to obtain a stable solution with high accuracy both for polar and nonpolar telescope locations. This result means that if we obtain at the stage of the direct problem for a telescope positioned at any point on the Moon the selenographic coordinates x_1, y_1 using the LRPs τ^c, ρ^c, σ^c calculated from the analytical theory [6], then, at the stage of the inverse problem, we can introduce in Eq. (2) the coordinates x_1, y_1 as observed coordinates and thus obtain the LRP values τ^o, ρ^o, σ^o with an error of less than 0.1″: $|\tau^o - \tau^c| < 0.1$ ms, $|\rho^o - \rho^c| < 0.1$ ms, $|\rho^o - \rho^c| < 0.1$ ms,

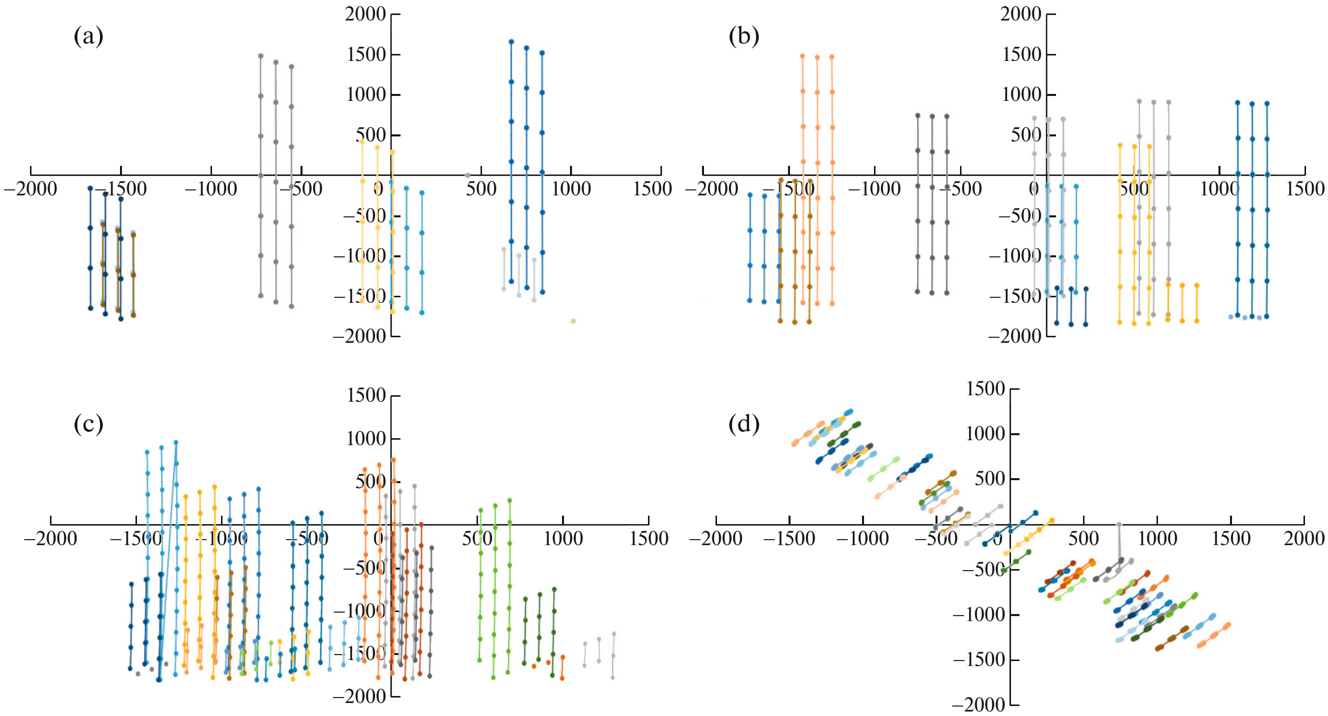

Fig. 4. The star tracks obtained by solving the direct problem, for stars in the field of view of the AZT, the selenographic coordinates of which were calculated with a step of 15 min during the first three hours of two lunar days. The points indicate the time of recording of a given star. The lines are the star tracks. (a) Star observations at the equator (latitude 0°). In this case, there are only 10 stars at the equator, with a maximum of 6 recordings over 3 h. (b) At latitude 30°. In this case, 11 stars are recorded, and there are 8 new appearances. (c) At latitude 60°. In this case, 14 stars are observed, each being recorded a maximum of 10 times. Of these, one star was recorded only once. (d) At the pole (latitude 90°). In this case, due to the slow motion of the stars, all the points recorded over the three hours of observations merged in fact into one big spot. Here, all the stars that fell within the FVT do not leave its visibility field.

and $I|\sigma^o - \sigma^c| < 0.1$ ms. Consequently, the gradient method as such does not introduce errors in the solution of Eq. (2) within the accuracy required for the experiment for all the simulated positions of the telescope.

The goal of the present simulation is to investigate the dependence of the accuracy of the LRP determinations on the latitude of the observation site in those cases where the values x_s^o, y_s^o, obtained from observations, are introduced at the stage of solving the inverse problem. The values x_s^o, y_s^o may also contain measurement errors and, importantly, differences in the parameters of the real Moon from the model assumed in the LPL theory. In other words, to what extent will the LRP values be sensitive to changes in the measured selenographic coordinates? At least, when modeling the polar observations, we showed [5, 6] that the longitudinal libration is absolutely not sensitive to observations and that measurement errors in the coordinates multiply the error in the latitude and node by $\sqrt{2}$. But what happens at other latitudes? We need to determine such a location of the AZT site where all the three LRPs equally noticeably react to any changes in the measured coordinates. In the case of good measurement accuracy, the discrepancies between the observed coordinates x_s^o and y_s^o and those calculated theoretically from the approximate model, x_s^c and y_s^c, will reflect their sensitivity only to factors unaccounted for in the model and thus provide information both to refine the model parameters and to improve the model itself.

To conduct the analysis, we performed calculations of the direct and inverse problems for AZTs positioned at different latitudes. In the case of an azimuthal mounting of the telescope, a natural challenge arises, as shown in Section 3. As the latitude of the observation site decreases, the velocity of motion of stars in the FVT increases. This happens because the AZT participates in two types of motion: slow precessional motion and fast diurnal motion. This, in turn, creates difficulties with the synchronization of observations at different latitudes, which is needed to measure the LRPs at one and the same time of observation.

Moreover, if we had decided to observe during a lunar day all the stars that fall within the FVT at the latitude of the observation site, we would have had to generate an enormous amount of stars and store their generated coordinates in computer memory. This would have required huge resources of external memory and, no less importantly, would have led to considerable expenditure of CPU time, which poses, at the stage of search modeling, a substantial obstacle to the analysis of results. If, for example, we took a step of coordinate recording at 1 h, it turned out that at low latitudes, we could at best record one appearance of a star in 1 h of observation (Fig. 3). However, at northern latitudes, the recording of stars can be carried out once every 1–2 days since many stars do not leave the FVT for as long as several months.

Therefore, in order to obtain a larger number of measurements per one recording of stars and reduce the CPU time per calculation, we found a simple yet not fully universal way. This approach relies on the fact that according to our simulations, most stars at latitudes below 75° are visible through a telescope for no more than three hours. During these three hours, we calculated the coordinates every 15 min. When the allotted time expired, we abruptly changed the current time by the duration of the lunar day, i.e., by 27.3 days, and again observed the same stars, which reappeared in the field of the AZT after such a rapid artificial diurnal rotation of the Moon. As a result, we conducted a triple series of observations of three hours each, recording every 15 min the coordinates of the same stars in each series (Fig. 4).

In fact, this regime is due to the described feature of observation of stars in the case of an azimuthal mounting of the telescope. To obtain the desired result, we had to simulate a process that can hardly be reproduced in reality—seven telescopes positioned at different latitudes simultaneously record the coordinates of a different number of stars for a long time. In order to maintain simultaneity and capture stars at southern latitudes, we need to take records at least every 15 min and save each measurement in RAM or in files. In one lunar day for one telescope only, provided that 20–40 stars appear in its field of view per month, the number of these measurements will be about 80000. However, in one month, one star (at southern latitudes) will yield 2–3 measurements, which, naturally, is not enough for statistics. Therefore, we had to continue the "observation" process for at least three lunar days. As a result, for one telescope, the calculation took 80–100 min, depending on the number of stars; most of the time was spent on working with files; and it is not possible to store huge data streams in RAM.

It should be noted that we, naturally, had no ready-made simulation algorithm. We were developing it in the course of studying the not always readily comprehensible behavior of stars; therefore, it made little sense to spend hours waiting for the result in order to analyze the solution and correct the algorithm. When we figured out why stars at different latitudes behave as shown in Fig. 4, we were able to develop the described observation regime, i.e., three hours of measurements at the beginning of each month, which allowed us to solve the posed problem, i.e., to identify the sensitivity of the LRPs to the measured selenographic coordinates of stars at different latitudes. This regime, or "schedule," of observations cannot be realized in practice—it is only the computer that can make leaps of 27 days in an instant. However, it is this method of recording stellar coordinates and solving the corresponding inverse problem that allowed us to solve the problem within a reasonable time.

By applying this observation technique, we were able, firstly, to record a large number of measurements of stars as they crossed the FVT and, secondly, to considerably reduce the calculation time. For all latitudes, we obtained the necessary values of both the coordinates and LRPs at the same time points, which is an important condition for an adequate analysis of the results.

Thus, we performed calculations for seven possible latitudes of the telescope site: from 0° to 90° with an interval of 15°. We showed that the longitude of the observation site affects only the time of appearance of the star in the FVT but does not affect the quality of the observations. Therefore, we confined ourselves to considering the positioning of telescopes at the first meridian, where their selenographic longitude is always zero.

From the analysis of the tracks presented in Fig. 4, we see that, at the southern latitudes, the number of points of recording is smaller than at the middle and northern latitudes. At latitude 90° (the north pole), we see segments of the spirals of almost all the chosen stars—during the observation time, they circle around the center of the telescope, moving very slowly away from it as the lunar pole precesses; over three hours of observations, the measurement points differ very little from one other, merging together to create a "blotchy" feeling.

In order to test the sensitivity of the LRPs to changes in the selenographic coordinates of the stars, we randomly changed by ±10 ms the values of x_s^c and y_s^c that were obtained from the direct problem. Then, we introduced the changed data into the solution of the inverse problem and checked how the three LRPs react to the changes in the "observed" coordinates. As a result, we obtained for each star the LRPs at all the times of coordinate recording.

A series of plots (Fig. 5) that demonstrate certain errors (O–C), i.e., differences between the original LRP value, from which x_s^c and y_s^c were calculated, and the one obtained from the changed selenographic coordinates, underpinned our analysis of the simulation results. Figure 5 allows us to compare how the errors (O–C) change, depending on the observation latitude for all the three LRPs.

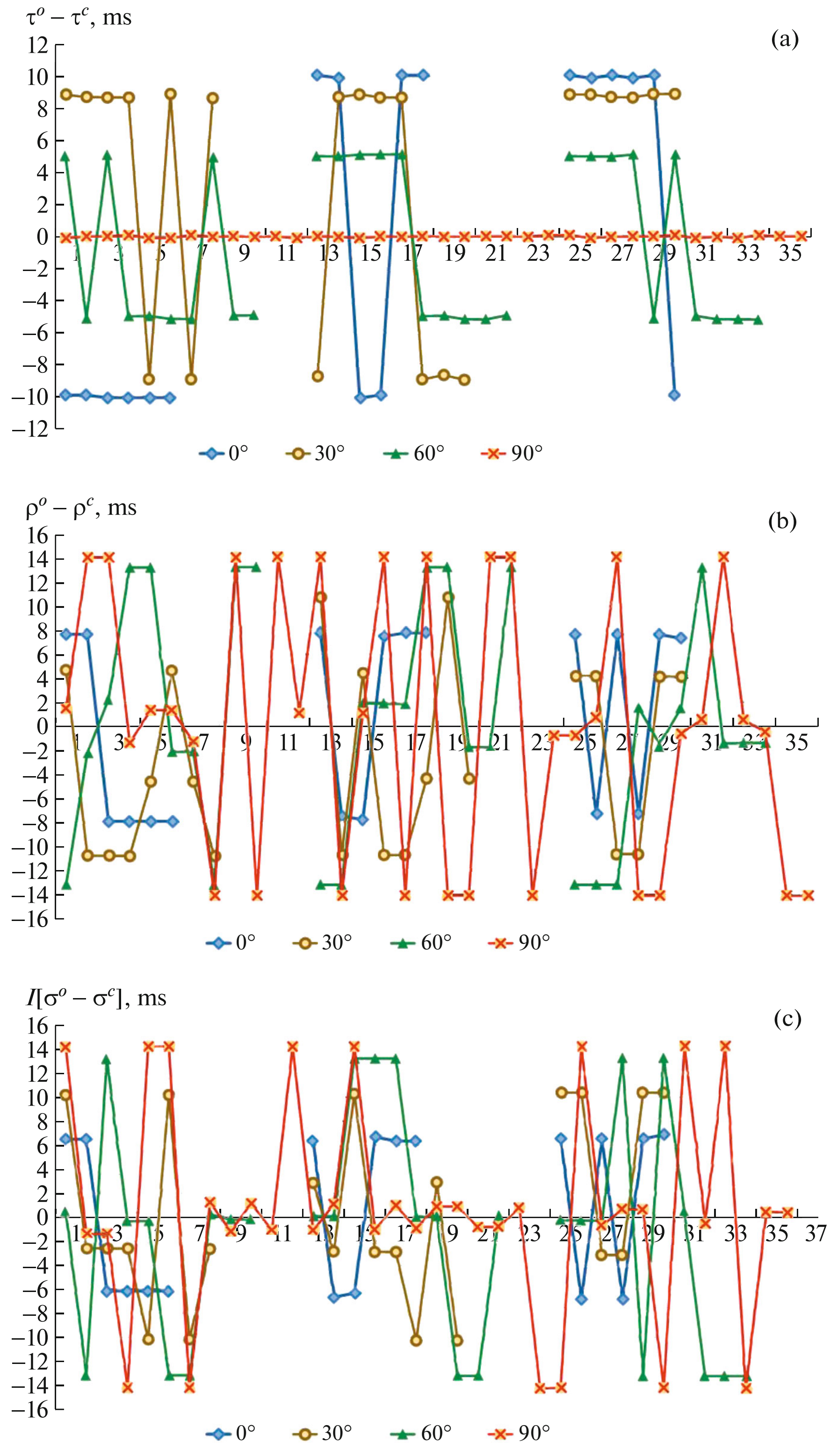

Fig. 5. Plots of (O–C) in the LRP determinations from one observation at different latitudes of the AZT site—at different times, the "observed stellar coordinates" x_s and y_s experienced random variations within ±10 ms. The abscissa axis shows the ordinal number of each star measurement, which occurs 15 min after the previous one for all the three series of measurements.

Having analyzed the residual differences at different latitudes, we can draw the following conclusions.

1. The maximum sensitivity of ξ in the LRPs to changes of ε in the selenographic coordinates x_s and y_s is obtained for ρ and σ from observations at the pole: $\xi = \sqrt{2}\varepsilon$, and for τ at the equator $\xi = \varepsilon$. The longitudinal libration cannot at all be determined from polar observations.

2. Conversely, equatorial observations provide not only good determinations of the longitude libration τ but also give a possibility to determine the other two parameters (the libration angles in the tilt ρ and node σ) although they are less sensitive to changes in the selenographic coordinates than in the northern latitudes. If the aim of the observation is to study the longitude libration τ, then AZTs should preferably be placed at a latitude of $\pm 3°$ (to avoid shielding by the Earth, if T is placed close to the first meridian).

3. At a latitude of 30°–45°, all the three LRPs have virtually the same error $\xi \cong \varepsilon$, which, in our opinion, makes these latitudes optimal for positioning a telescope capable of determining all the three LRPs, which equally respond to variations in selenographic coordinates.

4. Predictably, the errors (O–C), obtained by averaging over all the stars at the same time of observation, virtually shrink to zero at all the latitudes due to the random nature of the changes introduced by us at any point in time. This result necessitates such a schedule of future observations that would ensure that as many stars as possible fall within the FVT at the same time. If the changes in the observed coordinates are not random but functional, namely, associated, e.g., with applying a flawed model of the Moon's rotation, the average plot might also develop both systematic shifts and periodic variations, which in turn may serve as an observational groundwork for refining the model parameters.

5. A relative "drawback" of nonpolar AZT observations, apart from the lack of suitable conditions for the measuring equipment and partial shielding of the stars by the Sun, is the relatively high velocity of stars in the FVT, which may interfere with both the recording and identification of stars. However, bearing in mind that the Moon rotates at a velocity slower by a factor of 27 than the Earth, we can estimate that the velocity of rotation of stars from observations obtained at the lunar equator corresponds to that of stars at a latitude of ~88° on the Earth. It is very likely that a technology of terrestrial observations with such velocities already exists, and if the schedule of forthcoming observations is planned very carefully, there should not be any issues with the recording and identification of the observed objects.

5. CONCLUSIONS

The modern level of ongoing and planned studies of the Moon shows a high accuracy of observations and a variety of observational methods, among which an important place belongs, due to the specific conditions on the Moon, to the study of LPLs. This necessitates the development of new methods for analyzing large arrays of high-precision data on LRP observations and extracting from them the maximum amount of useful information about the Moon. In this context, the authors' experience of conducting a computer simulation of observations collected using a telescope placed on the lunar surface shows that this kind of experiment opens up new possibilities for revealing subtle effects in the lunar rotation, which, in turn, will allow scientists to delve into the intricate structure of the Moon's interior.

The computer simulation was carried out in two stages. The first one was the *direct simulation problem*, i.e., the calculation of the selenographic coordinates of stars on the basis of the selected dynamic model of the lunar body and the LRPs calculated from the analytical lunar rotation theory constructed for the model of a solid Moon. The second stage was the *inverse simulation problem*, i.e., the selenographic coordinates calculated at the first sage were then used to calculate the LRPs and analyze the residual differences in comparison with the original data. The gradient method applied for the inverse problem gives a high degree of accuracy. The same stage included the introduction of controlled errors in the "observed" coordinates, followed by analysis of the sensitivity of the resulting LRPs to these errors. The results of the inverse problem set the foundation for subsequent stages in the research, which include the solving of the inverse LPL problem, i.e., using the residual differences to refine the characteristics of the Moon's internal structure.

This paper describes a technique for computer simulation of observations collected with a lunar AZT, which can be placed at any point on the lunar surface. A substantiation is given for the criteria for determining the efficiency of the LRP observations. Visual materials are presented, which served as a basis for analyzing the research results.

The analysis of the simulated residual differences led to the following conclusion. In order to determine the libration angles in the tilt and node, it will be efficient to place the AZT at the lunar pole. However, determining the libration in longitude will require a second telescope, which should preferably be placed near the equator at a latitude of $\pm 3°$.

One telescope can be enough if placed at a latitude of 30°–45°, where all the three LRPs will be available for determination and equally sensitive to any variations in the measured selenographic coordinates. However, the nonpolar positioning of the telescope will require additional engineering research to ensure the operability of the measuring equipment as well as careful elaboration of the stellar identification techniques and meticulous scheduling of the observations.

FUNDING

This work was supported in part by the Russian Science Foundation, project no. 20-12-00105 (within which a method for data analysis was developed and numerical calculations were performed). The work was carried out in accordance with the Russian Government Program of Competitive Growth of Kazan Federal University. This work was supported in part by the scholarship of the President of the Russian Federation for young scientists and postgraduate students, no. SP-3225.2018.3, and the Foundation for the Advancement of Theoretical Physics and Mathematics "BASIS."

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Translated by A. Kobkova