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STRUCTURAL MECHANICS AND STRENGTH OF FLIGHT VEHICLES

An Investigation into the ASTM E756-05 Test Standard Accuracy on Determining the Damping Properties of Materials in Tension-Compression

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Abstract—The features of identifying the damping properties of viscoelastic materials caused by their dependence on a large number of external factors are discussed. The main propositions of the international standard on the experimental method for determining the damping properties of viscoelastic materials under cyclic tension–compression are analyzed. A refined finite element model of the dynamic behavior of the Oberst beam with a damping layer of a viscoelastic material with accounting of the transverse shear and compression is constructed. A numerical analysis of the error in determining the damping properties of a viscoelastic material under tension–compression caused by the neglect of transverse shear and compression strains arising in the layer of the material being tested is carried out.

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INTRODUCTION

Vibration damping via viscoelastic materials is a widely used and technologically simple method for solving the problems of sound and vibration protection of structural elements in aircraft, automotive, shipbuilding, and other industries. Achieving the desired results in the absorption of harmful vibrational energy is possible in various ways. However, it is very problematic to do with a minimal increase in the mass of the structure, which is especially important for the aerospace industry. Therefore, it is an urgent task.

The effectiveness of any damping material is determined by the product of its elastic modulus E by the loss coefficient η [1]. Moreover, the latter values can vary in different directions. Instead of the coefficient η, one can use the logarithmic decrement of vibrations $\delta = \eta\pi$. Elastomers based on rubber or thermoplastics based on bitumen with various fillers are often used as damping materials. To increase the viscoelastic properties in the operating temperature range, fillers, fibers, limestone, clay, tackifier, etc. are used as fillers [2–4].

The determination of the damping properties of viscoelastic materials is associated with significant difficulties. This is primarily due to the fact that damping properties are functions of a significant number of factors, namely, temperature, oscillation frequencies, strain level, modulus of elasticity, and preliminary force loading. In turn, the elastic modulus of such materials can also substantially depend on temperature, vibration frequency, and strain level. These facts greatly complicate the process of identifying the damping properties of a material from experimental results.

To date, there are several ways of the experimental investigation of damping properties [1, 5–7]. However, the most authoritative is the American standard ASTM E756-05 [8], which is based on the results of many years research on the damping of mechanical vibrations and experimental methods for identifying the damping properties of viscoelastic materials published in a fundamental monograph [1].

EXPERIMENTAL METHOD FOR DETERMINING THE DAMPING PROPERTIES OF VISCOELASTIC MATERIALS IN ACCORDANCE WITH ASTM E756-05

In accordance with the standard [8], the experimental method for determining the damping characteristics of a material under tension-compression is based on a comparative assessment of the dynamic behavior of cantilevered beam-type specimens without coating (Fig. 1a) and with coatings in the form of layers of the tested viscoelastic material (Figs. 1b and 1c). The latter two variants of the experimental samples are identical in terms of identifying the viscoelastic properties of the test material.

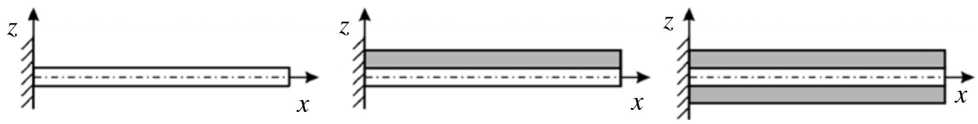

Fig. 1. Experimental samples of the beam type: (a) homogeneous beam (base); (b) a beam with one layer of damping material (the Oberst beam); (c) a beam with two layers of damping material.

On the basis of the logic of beam's static behavior (Figs. 1b and 1c) under their bending vibrations in the layers of the damping material only tensile-compression strains should arise (Fig. 2). It makes possible to determine the damping properties of a viscoelastic material from experiment data under such strain conditions.

Fig. 2. The physical picture of vibration damping: (a) undeformed state; (b) deformed state.

Studying the resonance vibrations of beams (Figs. 1b and 1c) and their description of the half-width spectral curve [1] for various modes, we can obtain the dependences of the elastic modulus E and the loss coefficient η on the oscillation frequency. Placing the beam system under investigation in a thermally insulated chamber allows one to obtain the dependence of these parameters on temperature.

In accordance with the assumptions accepted, the classical Bernoulli–Euler rod model is used to describe the deformation mechanics of the specimens being used (Figs. 1b and 1c), which does not take into account the transverse shear and compression of the viscoelastic layer. This assumption made it possible on the basis of reduction method [9, 10] to obtain very simple analytical expressions for calculating the elastic modulus E and the loss coefficient η for the viscoelastic material being tested from the experimental results.

However, it should be noted that the frequency range stated in the standard [8] for assessing the damping properties of viscoelastic materials covers frequencies from 50 to 5000 Hz. It is natural to assume that, at high vibrational frequencies, significant cyclic accelerations and, as a result, inertial forces can arise in the layers of viscoelastic material. They can lead to the appearance of significant cyclic strains and transverse compression stresses σ_z in the layers, which are not taken into account in the model of dynamic deformation being used [8]. In this case, the cyclic transversal compression strains ε_z will also be sources of energy absorption, which can distort the assessment of the damping properties of the test material under tension–compression. In this paper, a refined model of the dynamic deformation of the Oberst beam (Fig. 1b) is constructed for a possible estimate of the error in determining the damping properties of viscoelastic materials under tension–compression, introduced using a simplified model [1, 8].

REFINED MODEL OF DYNAMIC DEFORMATION OF THE OBERST BEAM

The Finite Element of the Oberst Beam, Geometric Dependencies

It is proposed using the finite element method [11, 12] to simulate the dynamic reaction of the Oberst beam with allowance for transverse shear and transverse compression strains of the damping layer. For this purpose, a two-layer finite element with ten degrees of freedom was developed (Fig. 3): layer 1 operates under the Kirchhoff hypothesis; the material of damping layer 2 is in a plane stress state and it is considered isotropic. The independent nodal parameters of the element are displacements u_i, w_i $(i = 1, 2, 3, 4)$ and rotation angles φ_1, φ_3 of the cross-sections of layer 1. They are represented by a vector $\mathbf{r}^{(e)} = \{u_1\ w_1\ \varphi_1\ u_2\ w_2\ u_3\ w_3\ \varphi_3\ u_4\ w_4\}$.

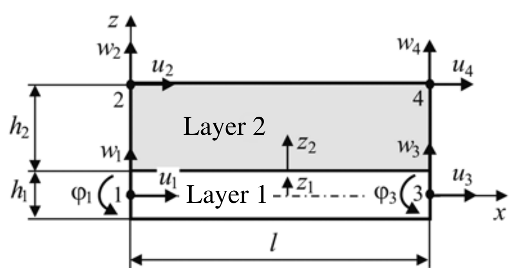

Fig. 3. The finite element of the Oberst beam.

The displacements $u^{(1)}$, $w^{(1)}$ of an arbitrary point located on the axis of layer 1 are approximated by the dependencies

$$u^{(1)} = \{H_1\ 0\ 0\ H_2\ 0\ 0\}^{\mathrm{T}} \mathbf{r}^{(1)};\ w^{(1)} = \{0\ N_1\ N_2\ 0\ N_3\ N_4\}^{\mathrm{T}} \mathbf{r}^{(1)} \tag{1}$$

with the nodal displacement vector $\mathbf{r}^{(1)} = \{u_1\ w_1\ \varphi_1\ u_3\ w_3\ \varphi_3\}$ and functions

$$H_1 = 1 - x/l;\ H_2 = x/l;\ N_1 = 1 - 3x^2/l^2 + 2x^3/l^3;\ N_2 = x - 2x^2/l + x^3/l^2;$$

$$N_3 = 3x^2/l^2 - 2x^3/l^3;\ N_4 = -x^2/l + x^3/l^2 .$$

The deformation $\varepsilon_x^{(1)}$ at a point located at a distance z_1 from the axis of layer 1 is determined by the geometric dependence

$$\varepsilon_x^{(1)} = d\left(u^{(1)} - \varphi^{(1)} z_1\right)/dx = d\left(u^{(1)} - w'^{(1)} z_1\right)/dx = u'^{(1)} - w''^{(1)} z_1,$$

in view of dependence (1), the latter one can be represented as

$$\varepsilon_x^{(1)} = \left(\mathbf{B}^{(1)}\right)^{\mathrm{T}} \mathbf{r}^{(1)}, \tag{2}$$

where $\mathbf{B}^{(1)} = \{H_1'\ 0\ 0\ H_2'\ 0\ 0\} - \{0\ N_1''\ N_2''\ 0\ N_3''\ N_4''\} z_1$.

The displacements $u^{(2)}$, $w^{(2)}$ of an arbitrary point of layer 2 with x, z_2 coordinates are represented by the bilinear dependencies

$$u^{(2)} = \left[\left(u_1 - \varphi_1 h_1/2\right)(1-\bar{x}) + \left(u_3 - \varphi_3 h_1/2\right)\bar{x}\right]\left(1-\bar{z}_2\right) + \left[u_2(1-\bar{x}) + u_4\bar{x}\right]\bar{z}_2; \tag{3}$$

$$w^{(2)} = \left[w_1(1-\bar{x}) + w_3\bar{x}\right](1-\bar{z}_2) + \left[w_2(1-\bar{x}) + w_4\bar{x}\right]\bar{z}_2 \quad \left(\bar{x} = x/l,\ \bar{z}_2 = z_2/h_2\right). \tag{4}$$

Strains $\varepsilon_x^{(2)}$, $\varepsilon_z^{(2)}$ and shear angle $\gamma_{xz}^{(2)}$ of layer 2 are

$$\varepsilon_x^{(2)} = \partial u^{(2)}/\partial x; \quad \varepsilon_z^{(2)} = \partial w^{(2)}/\partial z_2; \quad \gamma_{xz}^{(2)} = \partial w^{(2)}/\partial x + \partial u^{(2)}/\partial z_2. \tag{5}$$

Taking into account approximations (3) and (4), dependences (5) can be represented as

$$\boldsymbol{\varepsilon}^{(2)} = \begin{Bmatrix} \varepsilon_x^{(2)} \\ \varepsilon_z^{(2)} \\ \gamma_{xz}^{(2)} \end{Bmatrix} = \left[B^{(2)}\right]\mathbf{r}^{(e)} = \left[\left[B_1^{(2)}\right] \mid \left[B_2^{(2)}\right]\right]\begin{Bmatrix} \mathbf{r}_1^{(e)} \\ \mathbf{r}_2^{(e)} \end{Bmatrix}, \tag{6}$$

where

$$\left[B_1^{(2)}\right] = \left[\begin{array}{c|c|c|c|c} -\left(1-\bar{z}_2\right)/l & 0 & h_1\left(1-\bar{z}_2\right)/(2l) & -\bar{z}_2/l & 0 \\ \hline 0 & -\left(1-\bar{x}\right)/h_2 & 0 & 0 & \left(1-\bar{x}\right)/h_2 \\ \hline -(1-\bar{x})/h_2 & -\left(1-\bar{z}_2\right)/l & \left(1-\bar{x}\right)h_1/\left(2h_2\right) & \left(1-\bar{x}\right)/h_2 & -\bar{z}_2/l \end{array}\right], \tag{7}$$

$$\left[B_2^{(2)}\right] = \left[\begin{array}{c|c|c|c|c} \left(1-\bar{z}_2\right)/l & 0 & -h_1\left(1-\bar{z}_2\right)/(2l) & \bar{z}_2/l & 0 \\ \hline 0 & -\bar{x}/h_2 & 0 & 0 & \bar{x}/h_2 \\ \hline -\bar{x}/h_2 & \left(1-\bar{z}_2\right)/l & \bar{x}h_1/(2h_2) & \bar{x}/h_2 & \bar{z}_2/l \end{array}\right], \tag{8}$$

$$\mathbf{r}_1^{(e)} = \left\{u_1\ w_1\ \varphi_1\ u_2\ w_2\right\}, \quad \mathbf{r}_2^{(e)} = \left\{u_3\ w_3\ \varphi_3\ u_4\ w_4\right\}.$$

By setting the relative coordinates $\bar{z}_2$ and $\bar{x}$ in a certain way and knowing the vector of nodal displacements $\mathbf{r}^{(e)}$, we can calculate the deformations at any point of the damping layer of the finite element.

Construction of a Stiffness Matrix, Damping Matrix, and Mass Matrix of a Finite Element

The stiffness matrix, damping matrix, and mass matrix of the finite element of the Oberst beam are obtained by summing the corresponding matrices of layers formed relative to the vector $\mathbf{r}^{(e)}$:

$$\left[K^{(e)}\right] = \left[K^{(1)}\right] + \left[K^{(2)}\right]; \quad \left[C^{(e)}\right] = \left[C^{(1)}\right] + \left[C^{(2)}\right]; \quad \left[M^{(e)}\right] = \left[M^{(1)}\right] + \left[M^{(2)}\right]..$$

To obtain the stiffness matrix $\left[K^{(1)}\right]$ of layer 1, the matrix $\left[K_0^{(1)}\right]$ is taken as the initial one which is constructed with respect to nodal displacements $\mathbf{r}^{(1)}$ based on geometric dependence (2):

$$\left[K_0^{(1)}\right] = \left[\begin{array}{c|c|c|c|c|c} EF_1/l & 0 & 0 & -EF_1/l & 0 & 0 \\ \hline 0 & 12EI_1/l^3 & 6EI_1/l^2 & 0 & -12EI_1/l^3 & 6EI_1/l^2 \\ \hline 0 & 6EI_1/l^2 & 4EI_1/l & 0 & -6EI_1/l^2 & 2EI_1/l \\ \hline -EF_1/l & 0 & 0 & EF_1/l & 0 & 0 \\ \hline 0 & -12EI_1/l^3 & -6EI_1/l^2 & 0 & 12EI_1/l^3 & -6EI_1/l^2 \\ \hline 0 & 6EI_1/l^2 & 2EI_1/l & 0 & -6EI_1/l^2 & 4EI_1/l \end{array}\right];$$

$$EF_1 = E_1 b h_1, \quad EI_1 = E_1 b h_1^3/12.$$

For transition to the matrix $\left[K^{(1)}\right]$ relative to the vector $\mathbf{r}^{(e)}$ we use the transformation

$$\left[K^{(1)}\right]=\left[L^{(1)}\right]^{\mathrm{T}}\left[K_0^{(1)}\right]\left[L^{(1)}\right],$$

where $\left[L^{(1)}\right]$ is the matrix of the relationship of the vector $\mathbf{r}^{(1)}$ with the vector $\mathbf{r}^{(e)}$:

$$\mathbf{r}^{(1)}=\left[L^{(1)}\right]\mathbf{r}^{(e)};\ \left[L^{(1)}\right]=\begin{bmatrix}[S] & 0\\ 0 & [S]\end{bmatrix};\ [S]=\left[\begin{array}{c|c|c|c|c}1&0&0&0&0\\ \hline 0&1&0&0&0\\ \hline 0&0&1&0&0\end{array}\right]. \tag{9}$$

The matrix $\left[K^{(2)}\right]$ can be constructed using dependence (6) and the stiffness matrix $\left[D^{(2)}\right]$ of layer 2 material:

$$\left[K^{(2)}\right]=b\int_0^{h_2}\int_0^{l}\left[B^{(2)}\right]^{\mathrm{T}}\left[D^{(2)}\right]\left[B^{(2)}\right]dxdz_2; \tag{10}$$

$$\left[D^{(2)}\right]=\left[\begin{array}{c|c|c}E_2/\left(1-\nu_2^2\right) & E_2\nu_2/\left(1-\nu_2^2\right) & 0\\ \hline E_2\nu_2/\left(1-\nu_2^2\right) & E_2/\left(1-\nu_2^2\right) & 0\\ \hline 0 & 0 & G_2\end{array}\right].$$

Here E_2 and G_2 are the elastic moduli of the material under tension–compression and shear, respectively. We introduce transformation to new coordinates:

$$x=(1+\xi)l/2;\quad z_2=(1+\eta)h_2/2;\quad -1\le\xi\le1;\quad -1\le\eta\le1.$$

With this in mind, expression (10) takes the form

$$\left[K^{(2)}\right]=0.25bh_2l\int_{-1}^{1}\int_{-1}^{1}\left[B^{(2)}\right]^{\mathrm{T}}\left[D^{(2)}\right]\left[B^{(2)}\right]d\xi d\eta. \tag{11}$$

The integral in (11) can be found numerically. From expressions (6)–(8) and the introduced transformation of coordinates, it follows that the product $\left[B^{(2)}\right]^{\mathrm{T}}\left[D^{(2)}\right]\left[B^{(2)}\right]$ depends quadratically on the coordinates ξ, η. In this case, to calculate the integral accurately, we use the Gaussian quadrature formula with two points along each of the coordinates ξ, η [13, 14]:

$$\left[K^{(2)}\right]=0.25bh_2l\sum_{m=1}^{2}\sum_{n=1}^{2}\left[B^{(2)}\left(\xi_m,\eta_n\right)\right]^{\mathrm{T}}\left[D^{(2)}\right]\left[B^{(2)}\left(\xi_m,\eta_n\right)\right]P_mQ_n;$$

$$\xi_1=\eta_1=-0.57735;\quad \xi_2=\eta_2=0.57735;\quad P_1=P_2=Q_1=Q_2=1.$$

Let us proceed to the construction of damping matrices $\left[C^{(i)}\right]$ $(i=1,\ 2)$ of the element layers. The type of these matrices depends on the model of inelastic deformation of the material. In uniaxial stress state, the simplest of such models corresponds to the well-known Voigt–Thomson–Kelvin model [7, 15], with dependence of the following form:

$$\sigma=E\varepsilon+\beta\dot{\varepsilon}, \tag{12}$$

where σ, ε, $\dot{\varepsilon}$ are the normal stress, relative strain, and its rate of change in time t, respectively; E, β are the elastic modulus and viscosity coefficient of the material. The latter is related to the logarithmic decrement of vibrations δ of the material by the dependence [16] $\beta=E\delta/(\pi\omega)$, where ω is the circular frequency of deformation of the material. In this case, dependence (12) takes the form

$$\sigma = E\varepsilon + E\delta\dot{\varepsilon}/(\pi\omega). \quad (13)$$

We assume that the damping properties of the material of the element layers are constant (do not depend on the deformation amplitudes of these layers). Then, following expression (13), the damping matrix $\left[C^{(1)}\right]$ can be obtained by multiplying the stiffness matrix $\left[K^{(1)}\right]$ of layer 1 by the value $\delta_1/(\pi\omega)$, where δ_1 is the logarithmic decrement of vibrations of the material of this layer. The matrix $\left[C^{(2)}\right]$ is obtained from the stiffness matrix $\left[K^{(2)}\right]$ by replacing the matrix $\left[D^{(2)}\right]$ in it with a material damping matrix

$$\left[D_g^{(2)}\right] = \frac{1}{\pi\omega}\left[\begin{array}{c|c|c} E_2\delta_{2,\varepsilon}/\left(1-\nu_2^2\right) & E_2\delta_{2,\varepsilon}\nu_2/\left(1-\nu_2^2\right) & 0 \\ \hline E_2\delta_{2,\varepsilon}\nu_2/\left(1-\nu_2^2\right) & E_2\delta_{2,\varepsilon}/\left(1-\nu_2^2\right) & 0 \\ \hline 0 & 0 & G_2\delta_{2,\gamma} \end{array}\right],$$

where $\delta_{2,\varepsilon}$, $\delta_{2,\gamma}$ are the logarithmic decrement of vibrations of the material of layer 2 under tension–compression and shear, respectively.

To construct the matrices $\left[M^{(1)}\right]$ and $\left[M^{(2)}\right]$, we used the assumption that the kinetic energy of the element layers is determined only by the velocities $\dot{w}^{(i)}$ $(i = 1, 2)$.

Formation of a System of Resolving Equations

Let us suppose that the load is reduced to the nodes of the element and is represented by a vector $\mathbf{P}^{(e)}(t)$. Then the equations of motion of the element will have the form

$$\left[M^{(e)}\right]\ddot{\mathbf{r}}^{(e)} + \left[C^{(e)}\right]\dot{\mathbf{r}}^{(e)} + \left[K^{(e)}\right]\mathbf{r}^{(e)} = \mathbf{P}^{(e)}(t).$$

Combining these equations in the directions common to adjacent elements of nodal displacements, we arrive at the equations of motion of the finite element model of the beam

$$[M]\ddot{\mathbf{r}} + [C]\dot{\mathbf{r}} + [K]\mathbf{r} = \mathbf{P}(t). \quad (14)$$

Here $[M]$, $[C]$, $[K]$, $\mathbf{r}$, $\mathbf{P}(t)$ are the mass matrix, damping matrix, stiffness matrix, nodal displacement vector, and external nodal forces vector of the model, respectively.

Let us consider the resonance vibrations under the action of a load $\mathbf{P}(t) = \mathbf{P}_0 e^{ipt}$ with the amplitude $\mathbf{P}_0$ and the frequency $p = \omega_j$, where ω_j is one of the frequencies of free oscillations of the beam. In this case, the oscillations will occur in the mode $\mathbf{F}_j$ corresponding to the frequency ω_j:

$$\mathbf{r} = s_j(t)\mathbf{F}_j, \quad (15)$$

where $s_j(t)$ is the normal coordinate. Substituting expression (15) and the load $\mathbf{P}(t) = \mathbf{P}_0 e^{ipt}$ into system (14) and applying the procedure of the Bubnov–Galerkin method after this, we obtain the equation with respect to the coordinate $s_j(t)$:

$$m_j\ddot{s}_j + c_j\dot{s}_j + k_j s_j = p_{0,j}e^{ipt} \quad (16)$$

with modal parameters

$$m_j = \mathbf{F}_j^{\mathrm{T}}[M]\mathbf{F}_j,\ \ c_j = \mathbf{F}_j^{\mathrm{T}}[C]\mathbf{F}_j,\ \ k_j = \mathbf{F}_j^{\mathrm{T}}[K]\mathbf{F}_j,\ \ p_{0,j} = \mathbf{F}_j^{\mathrm{T}}\mathbf{P}_0.$$

The solution to Eq. (16) will be sought in the form

$$s_j(t) = s_{0,j} e^{i(pt - \varphi_j)}, \tag{17}$$

where φ_j is the phase shift of the coordinate $s_j(t)$ relative to the load vector $\mathbf{P}_0 e^{ipt}$. Substituting expression (17) into Eq. (16) and then reducing the common multiplier e^{ipt}, we arrive at the system of equations

$$\left[\begin{array}{c|c} k_j - p^2 m_j & pc_j \\ \hline pc_j & -k_j + p^2 m_j \end{array}\right] \begin{Bmatrix} s_{a,j} \\ s_{b,j} \end{Bmatrix} = \begin{Bmatrix} p_{0,j} \\ 0 \end{Bmatrix},$$

where $s_{a,j} = s_{0,j} \cos\varphi_j$, $s_{b,j} = s_{0,j} \sin\varphi_j$. From here you can find the components $s_{a,j}$ and $s_{b,j}$, from which you can determine the amplitude $q_{0,j}$ and $\tan\varphi_j$:

$$s_{0,j} = \sqrt{s_{a,j}^2 + s_{b,j}^2}; \;\; \tan\varphi_j = s_{b,j} / s_{a,j}.$$

The frequencies ω_j and modes $\mathbf{F}_j$ of free vibrations of the beam can be determined from a system of homogeneous equations [17, 18]

$$\left([K] - \omega^2 [M]\right)\mathbf{F} = 0, \tag{18}$$

constituting the generalized eigenvalue problem and eigenvectors of the set of matrices $[K]$ and $[M]$. The eigenvalues are the squares of the frequencies ω_j, the eigenvectors are the modes $\mathbf{F}_j$. To solve problem (18), it is advisable to use the iteration method in the subspace [17, 19], which allows iterating at the same time several lower forms and frequencies.

THE RESULTS OF NUMERICAL INVESTIGATIONS

The proposed and implemented mathematical model of the dynamic behavior of the Oberst beam made it possible to study the stress and strain state of a viscoelastic layer under resonant vibrations. A cantilever beam (Fig. 1b) with artificially set damping layer parameters satisfying all the requirements of the standard [8] was adopted as an object of study.

The base (Fig. 1a) is made of steel and has the following geometric and physical characteristics: elastic modulus $E_1 = 2.1 \times 10^5$ MPa; density $\rho_1 = 7800$ kg/m^3; length L = 200 mm; width b = 10 mm; thickness $h_1 = 1$ mm. The damping properties of the base are not taken into account ($\delta_1 = 0$).

The damping layer of a viscoelastic material is characterized by the following parameters: elastic modulus $E_2 = 200$ MPa; Poisson's ratio $\nu_2 = 0.3$; density $\rho_2 = 2000$ kg/m^3; thickness $h_2 = 3$ mm. The logarithmic decrement of vibrations of the material under tension–compression, transverse compression and transverse shear are assumed to be the same: $\delta_x = \delta_z = \delta_{xz} = 1$.

A linear load $q(t) = q_0 \cos pt$ with an amplitude of $q_0 = 0.9$ N/m and a frequency p coinciding with one of the natural frequencies ω_j acts on the beam. The beam is divided into 100 finite elements of the same length.

Figure 4 shows the amplitudes of the stresses $\sigma_{x,0}$, $\sigma_{z,0}$, $\tau_{xz,0}$ in the middle surface of the damping layer under resonant vibrations of the beam in the seventh bending mode (vibration frequency f = 2015.54 Hz).

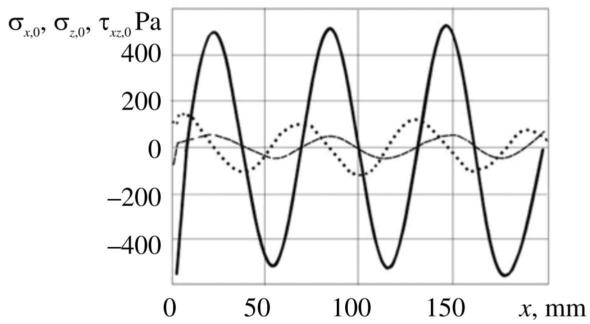

Fig. 4. The amplitudes of the stresses in the middle surface of the damping layer under resonant vibrations of the beam in the seventh mode:——— — $\sigma_{x,0}$; ----- — $\sigma_{z,0}$; ••••• — $\tau_{xz,0}$.

As it turned out, in the damping layer, in addition to the dominant stresses σ_x, transverse compression stresses σ_z arise. Furthermore, it is important to note, even more significant are the stresses τ_{xz}. Stresses σ_z and τ_{xz} constitute approximately 10 and 19 % of the stresses σ_x, respectively. As the frequency f increases, the contribution of stresses σ_z and τ_{xz} to the stress state of the layer becomes even higher. For example, in the case of vibrations of the beam under the tenth resonant mode (f = 4293.89 Hz), the stresses σ_z and τ_{xz} are approximately 22 and 33 % of the stress σ_x, respectively.

It is natural to assume that the arising transverse shear and compression stresses can significantly affect the damping of the beam vibrations as a whole and, accordingly, the evaluation of the damping properties of the viscoelastic material under tension-compression being tested.

Table 1 shows the mode numbers of free vibrations of the beam under study, the corresponding frequencies f, logarithmic decrement of vibrations (LDV) of the beam, caused by energy dissipation in the damping layer only due to stresses σ_x (LDV-1), under the combined action of stresses σ_z and τ_{xz} (LDV-2), as well as with the participation of all acting stresses (LDV-3). The last column of the table shows the miscalculation in the direction of overestimation in determining the damping properties of a viscoelastic material under tension–compression (LDV-2 / LDV-1). It can be seen that, at high vibrational frequencies, the rate of in the indicated miscalculation increases rapidly.

Table 1

Mode number	f, Hz	LDV-1	LDV-2	LDV-3	Miscalculation, %
1	17.04	0.13831	0.01416	0.15247	10.24
2	106.74	0.13686	0.01467	0.15152	10.72
3	298.79	0.13453	0.01528	0.14984	11.36
4	585.29	0.13128	0.01607	0.14735	12.24
5	967.12	0.12719	0.01702	0.14421	13.38
6	1443.96	0.12238	0.01815	0.14053	14.83
7	2015.54	0.11698	0.01947	0.13646	16.64
8	2681.45	0.11113	0.02103	0.13216	18.92
9	3441.15	0.10495	0.02288	0.12783	21.80
10	4293.89	0.09858	0.02509	0.12367	25.46
11	5238.66	0.09212	0.02778	0.11990	30.16
12	6274.08	0.08568	0.03108	0.11675	36.27
13	7398.32	0.07934	0.03517	0.11451	44.33
14	8608.89	0.07317	0.04030	0.11346	55.08

CONCLUSIONS

This paper does not claim to be a complete analysis of the errors in applying the standard [8] on the determination of the damping properties of viscoelastic materials under tension–compression in the frequency range from 50 to 5000 Hz. However, it allows us to make the following conclusions:

—the results presented indicate that neglecting the transverse shear and compression stresses in the layers of viscoelastic material can lead to significant miscalculation in the direction of overestimation in determining the damping properties under tension-compression;

—the dependences of the damping properties of viscoelastic materials in the sound frequency range given in [1] can be significantly distorted and are unacceptable for a reliable assessment of the effectiveness of vibration-absorbing coatings.

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