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AIRCRAFT AND ROCKET ENGINE DESIGN AND DEVELOPMENT

Modeling the Transformation of Hexactinal Folded Structure

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Abstract—A mathematical model is developed to study the transformation of hexactinal folded structures. Equations for calculation of the structure element pathways during transformation are obtained based on the vector analysis methods. An algorithm is presented together with a model problem solution.

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INTRODUCTION

Folded structure (FS) is a facet surface composed of typical recurrent fragments that may be unfolded into a plane. Each fragment is a combination of flat faces connected along the edges. Folded structures may serve as geometric basis of light cores for sandwich panels [1–4], sound absorbing structures [5–7], plates of heat exchangers [8], filters [9], radio scattering shells [10], compound-chain mechanisms.

If we imagine that the faces of the FS can turn around the edges connecting them, we may speak about transformation of the structure.

Mutual arrangement of the faces changes during transformation. In the initial position, all faces are in the same plane. In the final position, structure becomes either a compact block with faces completely adjacent to each other, either a block with faces in plane, or a cellular block (Fig. 1) [11].

So, transformation is a process of transition from one extreme state of the FS into the other ones involving the change of the density of the FS surface relief.

Each structure follows its own transformation principles. One has to know these principles when designing the manufacturing process for the forming of these structures and calculating parameters of equipment. The same goes for mechanisms (deployable frames for solar panels and antennas) that have deployment kinematics based on folded structures.

It is reasonable to describe transformation based on the principles of FS nodal points' displacement based on a trigger factor. A tailored approach of two nodes in the structure or a change of an angle between two edges may be a trigger factor.

Each folded structure has its own principles of nodal points' displacement. At the same time, the common principles for construction of transformation model, same for any structure, were developed in [12]. Simulation of transformation is based on construction of a vector model of a typical fragment. All the nodal points are defined by the system of interconnected vectors in a given coordinate system. This technique is applied in a range of works [13–17] and all the research in these papers deal with design of modified four-rayed structures.

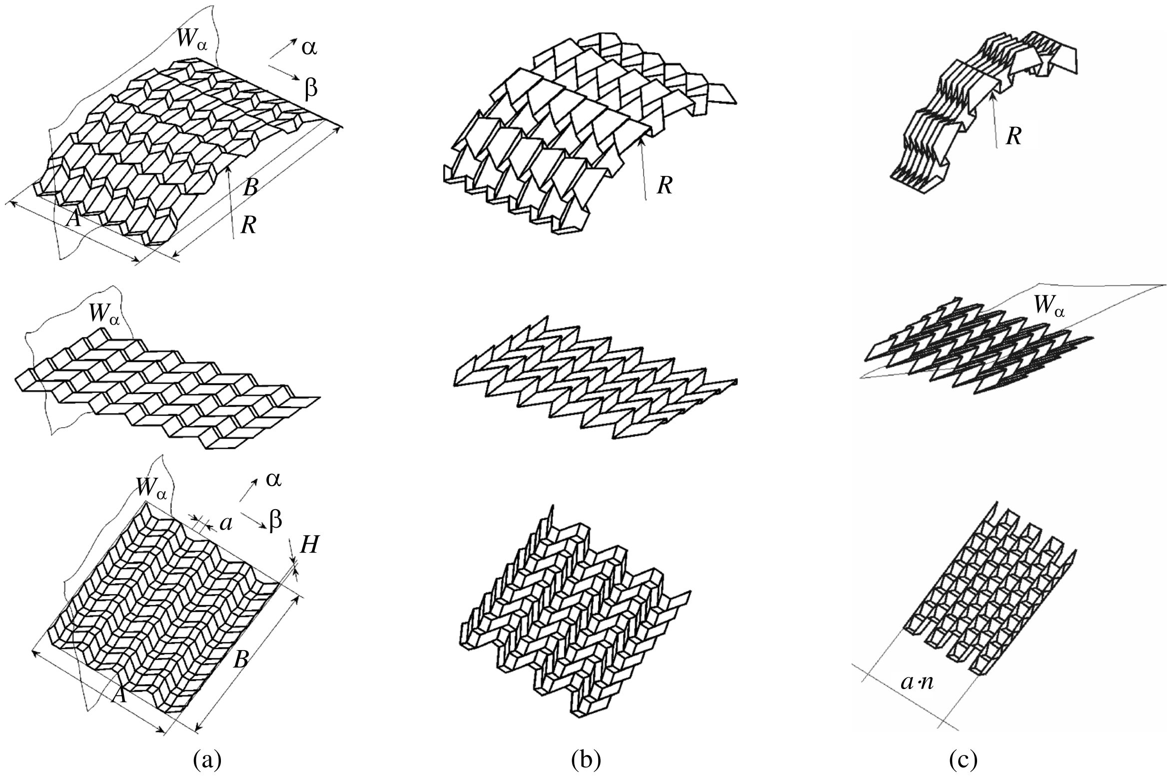

Fig. 1. Transformation of structure: (a)—flat state (developed view); (b)—transformation; (c)—final state.

Meanwhile, another group of folded structures, namely, hexactinal structures, is of great interest. These structures have a wider range of architectures and variants of future application. They are described in [18–20] together with methods of their synthesis.

The aim of this research is to create a vector model for transformation of hexactinal folded structures [18].

PROBLEM DESCRIPTION

Figure 2a demonstrates a hexactinal structure in variant A [17]. This structure has a pattern shown in Fig. 2b. It consists of periodically recurrent typical fragments (Fig. 2c)—elementary units. Each elementary unit consists of six interconnected triangles with edges between them. The elementary unit contains all the nesessary information about geometry of the structure. If the principles of transformation of this elementary unit are determined, it is possible to talk about solution of the problem for the entire structure.

To construct a vector model of the elementary unit, it should be put into x, y, z (Fig. 2d) coordinates system with the center in the point, where six edges intersect. Let us place vectors $\overline{r}_3^0$ along each edge. The developed view of such an elementary unit with vectors is demonstrated in Fig. 2d. Let us assume that vectors $\overline{r}_3^0$ and $\overline{r}_7^0$ stay in the plane O_{xy} during transformation. Rotation of vector $\overline{r}_7^0$ in the plane O_{xy} around the origin is a factor initiating transformation (Fig. 2e). Due to the fact that all vectors are interconnected by the edges, initiating movement of vector $\overline{r}_7^0$ leads to transformation of the entire unit. The termini of the vectors except for $\overline{r}_3^0$ and $\overline{r}_7^0$ are getting out of the plane. The problem will be solved, if the equations for the movement paths of vectors' termini are defined.

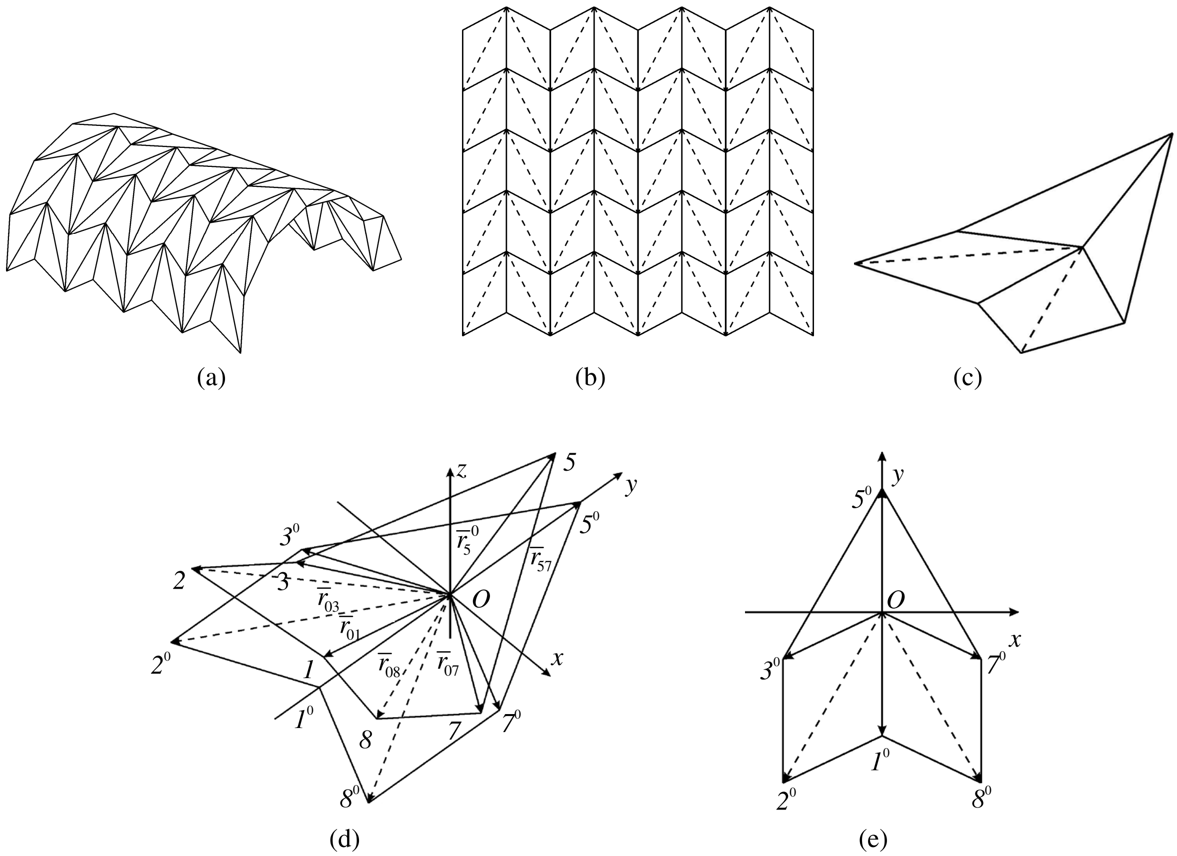

Fig. 2. Hexactinal folded structure: (a)—the structure; (b)—the developed view; (c)—the elementary unit; (d)—the vector model; (e)—the vector model on the plane (notations with superscripts are related to vectors' termini, e.g. 5^0 is related to $\overline{r}_5^0$).

Initial geometrical precondition for the equations deduction is the fact that the vector lengths (sides of the faces) as well as areas of triangular faces stay the same during transformation.

Thus, in the initial state, interconnected six triangles are located on the plane O_{xy}. Geometry of the triangles is determined by vectors $\overline{r}_1^0$, $\overline{r}_8^0$, $\overline{r}_7^0$, $\overline{r}_5^0$, $\overline{r}_3^0$, $\overline{r}_2^0$, which termini are specified in Fig. 2e. The triangles are located symmetrically relatively to the plane O_{xz}. Here and further in the text, zero superscripts mean that the vectors and their components are located in the plane O_{xy}. When the triangles move in the space O_{xyz}, vectors $\overline{r}_3$, $\overline{r}_7$ stay in the plane O_{xy}, i.e. $\overline{r}_3=\left\{r_{3,x},\ r_{3,y},\ 0\right\}$, $\overline{r}_7=\left\{r_{7,x},r_{7,y},0\right\}$. In the general case, $\overline{r}_3\neq\overline{r}_7$, however, $\left|\overline{r}_3\right|=\left|\overline{r}_7\right|$. Triangles in the plane O_{xy} initially have areas $S_{75}^0=S_{53}^0$; $S_{32}^0=S_{87}^0$; $S_{21}^0=S_{18}^0$.

With angular displacement of vectors $\overline{r}_3=\left\{r_{3,x},\ r_{3,y},\ 0\right\}$, $\overline{r}_7=\left\{r_{7,x},r_{7,y},0\right\}$ on the plane O_{xy}, vectors $\overline{r}_5$, $\overline{r}_2$, $\overline{r}_1$, and $\overline{r}_8$ are located in the space O_{xyz}.

Thus, given the values of projection of vectors $r_{3,x}$, $r_{3,y}$, $r_{7,x}$, $r_{7,y}$, it is necessary to find location of vectors $\overline{r}_5^0$, $\overline{r}_2^0$, $\overline{r}_1^0$, and $\overline{r}_8^0$ in the space O_{xyz}, if the modules of these vectors are initially known $\left|\overline{r}_5\right|=\left|\overline{r}_5^0\right|$; $\left|\overline{r}_2\right|=\left|\overline{r}_2^0\right|$; $\overline{r}_1=\left|\overline{r}_1^0\right|$; $\left|\overline{r}_8\right|=\left|\overline{r}_8^0\right|$.

DETERMINING THE AREAS OF TRIANGLES IN THE INITIAL STATE

Areas of the triangles on the plane O_{xy} are determined through the vector product of the vectors:

$$S_{75}^0 = S_{35}^0 = \left|\overline{r}_7^0 \times \overline{r}_5^0\right|/2 = \frac{1}{2}\begin{Vmatrix} i & j & k \\ r_{7,x}^0, & r_{7,y}^0, & 0 \\ r_{5,x}^0, & r_{5,y}^0, & 0 \end{Vmatrix} = \frac{1}{2}\left(r_{7,x}^0,\ r_{5,y}^0 - r_{7,y}^0,\ r_{5,x}^0\right).$$

Since $r_{5,x}^0 = 0$, then $S_{75}^0 = S_{35}^0 = r_{7,x}^0 r_{5,y}^0/2$.

Taking into account $r_{1,x}^0 = r_{1,z}^0 = r_{8,z}^0 = 0$ for areas $S_{18}^0 = S_{12}^0$, we get

$$S_{18}^0 = S_{12}^0 = \left|\overline{r}_1^0 \times \overline{r}_8^0\right|/2 = \frac{1}{2}\begin{Vmatrix} i & j & k \\ 0 & r_{1,y}^0 & 0 \\ r_{8,x}^0 & r_{8,y}^0 & 0 \end{Vmatrix} = r_{8,x}^0 r_{1,x}^0/2,$$

for areas $S_{32}^0 = S_{87}^0$, respectively,

$$S_{32}^0 = S_{87}^0 = \left|\overline{r}_3^0 \times \overline{r}_2^0\right|/2 = \frac{1}{2}\begin{Vmatrix} i & j & k \\ r_{3,x}^0 & r_{3,y}^0 & 0 \\ r_{2,x}^0 & r_{2,y}^0 & 0 \end{Vmatrix} = \left(r_{3,x}^0, r_{2,y}^0 - r_{3,y}^0, r_{2,x}^0\right)/2.$$

Thus, the areas of all triangles on the plane O_{xy} are defined through the coordinates of the vertices of the triangles.

Values and geometric shapes of these triangles stay the same, when vectors $\overline{r}_5$, $\overline{r}_2$, $\overline{r}_1$ and $\overline{r}_8$ move in space (Fig. 2c).

DETERMINING THE VECTORS POSITION IN SPACE

Determining the position of vector $\overline{r}_5$

Cosine of the angle between vectors $\overline{r}_5$ and $\overline{r}_3$ is determined by the scalar product of the vectors

$$\cos\left(\overline{r}_5, \overline{r}_3\right) = \frac{-\left(\overline{r}_5\ \overline{r}_3\right)}{\left|\overline{r}_5\right|\left|\overline{r}_3\right|} = \frac{-\left(r_{5,x}\, r_{3,x} + r_{5,y}\, r_{5,y} + r_{5,z}\, 0\right)}{\left|\overline{r}_5\right|\left|\overline{r}_3\right|}.$$

Here, the minus means that the angle between these two vectors is more than 90 degrees. Sine of the angle between these vectors is defined by the area of the triangle based on these vectors $\left|\overline{r}_5\right|\left|\overline{r}_3\right|\sin\left(\overline{r}_5, \overline{r}_3\right) = 2S_{53}^0$, since $S_{53} = S_{53}^0$, or

$$\cos\left(\overline{r}_5, \overline{r}_3\right) = \sqrt{1-\sin^2\left(\overline{r}_5, \overline{r}_3\right)} = \sqrt{1-\left(\frac{2\,S_{53}^0}{\left|\overline{r}_5\right|\left|\overline{r}_3\right|}\right)^2} = \frac{\sqrt{\left|\overline{r}_5\right|^2\left|\overline{r}_3\right|^2 - 4\left(S_{53}^0\right)^2}}{\left|\overline{r}_5\right|\left|\overline{r}_3\right|}.$$

Thus, comparing numerators of two latter equations for $\cos\left(\overline{r}_5, \overline{r}_3\right)$, we get

$$-\left(r_{5,x}\, r_{3,x} + r_{5,y}\, r_{3,y}\right) = a_1, \tag{1}$$

where $a_1 = \sqrt{\left|\overline{r}_5\right|^2\left|\overline{r}_3\right|^2 - 4\left(S_{53}^0\right)^2}$.

Similarly for $\overline{r}_7$ and $\overline{r}_5$,

$$-\left(r_{5,x}\, r_{7,x} + r_{5,y}\, r_{7,y}\right) = a_2\,, \tag{2}$$

where $a_2 = \sqrt{\left|\overline{r}_5\right|^2 \left|\overline{r}_7\right|^2 - 4\left(S^0_{75}\right)^2}$.

From Eqs. (1) and (2), we get a system of two equations to define $r_{5,x}$ and $r_{5,y}$

$$r_{3,x}\, r_{5,x} + r_{3,y}\, r_{5,y} = -a_1\,;$$

$$r_{7,x}\, r_{5,x} + r_{7,y}\, r_{5,y} = -a_2\,.$$

Solving these equations, we get

$$r_{5,x} = \frac{-a_1\; r_{7,y} + a_2\; r_{3,y}}{r_{3,x}\, r_{7,y} - r_{3,y}\, r_{7,x}}; \tag{3}$$

$$r_{5,y} = \frac{-a_2\; r_{3,x} + a_1\, r_{7,x}}{r_{3,x}\, r_{7,y} - r_{3,y}\, r_{7,x}}. \tag{4}$$

Keeping in mind that $\left|\overline{r}_5\right|^2 = r^2_{5,x} + r^2_{5,y} + r^2_{5,z}$, we get

$$r_{5,z} = \left(\left|\overline{r}_5\right|^2 - r^2_{5,x} - r^2_{5,y}\right)^{1/2}. \tag{5}$$

Vector $\overline{r}_5$ is determined: $\overline{r}_5 = \left\{r_{5,x},\, r_{5,y},\, r_{5,z}\right\}$.

In Eqs. (1) and (2), if $\left|\overline{r}_3\right| = \left|\overline{r}_7\right|$, $S_{53} = S_{75}$, then $a_1 = a_2$.

Similarly to Eqs. (1) and (2), considering vectors $\overline{r}_7$ and $\overline{r}_8$, we get

$$r_{7,x}\, r_{8,x} + r_{7,y}\, r_{8,y} = a_3\,, \tag{7}$$

where $a_3 = \sqrt{\left|\overline{r}_7\right|^2 \left|\overline{r}_8\right|^2 - 4\left(S^0_{78}\right)^2}$.

For vectors $\overline{r}_3$ and $\overline{r}_2$, we get

$$r_{3,x}\, r_{2,x} + r_{3,y}\, r_{2,y} = a_4\,, \tag{8}$$

where $a_4 = \sqrt{\left|\overline{r}_3\right|^2 \left|\overline{r}_2\right|^2 - 4\left(S^0_{32}\right)^2}$. For Eqs. (7) and (8), $a_3 = a_4$.

For $\overline{r}_2$ and $\overline{r}_1$,

$$r_{2,x}\, r_{1,x} + r_{2,y}\, r_{1,y} + r_{2,z}\, r_{1,z} = a_5\,, \tag{9}$$

where $a_5 = \sqrt{\left|\overline{r}_2\right|^2 \left|\overline{r}_1\right|^2 - 4\left(S^0_{21}\right)^2}$.

For vectors $\overline{r}_1$ and $\overline{r}_8$,

$$r_{1,x}\, r_{8,x} + r_{1,y}\, r_{8,y} + r_{1,z}\, r_{8,z} = a_6\,, \tag{10}$$

where $a_6 = \sqrt{\left|\overline{r}_1\right|^2 \left|\overline{r}_8\right|^2 - 4\left(S^0_{18}\right)^2}$. For Eqs. (9) and (10), $a_5 = a_6$.

Modules of vectors $\overline{r}_1$, $\overline{r}_2$, and $\overline{r}_8$ are known

$$\left|\overline{r_1}\right|^2 = r_{1,x}^2 + r_{1,y}^2 + r_{1,z}^2 ; \tag{11}$$

$$\left|\overline{r_2}\right|^2 = r_{2,x}^2 + r_{2,y}^2 + r_{2,z}^2 ; \tag{12}$$

$$\left|\overline{r_8}\right|^2 = r_{8,x}^2 + r_{8,y}^2 + r_{8,z}^2 . \tag{13}$$

So, we have seven equations (7)–(13) and nine unknown components of vectors $\overline{r_1}$, $\overline{r_2}$, $\overline{r_8}$.

When vector $\overline{r_3}$ and $\overline{r_7}$ are fixed in the plane O_{xy} with known components $\overline{r_3} = \left\{r_{3,x},\ r_{3,y},\ 0\right\}$, $\overline{r_7} = \left\{r_{7,x}, r_{7,y}, 0\right\}$, the line based on vectors $\overline{r_{32}}$, $\overline{r_{21}}$, $\overline{r_{18}}$, $\overline{r_{87}}$ may have two degrees of freedom. For the system of describing equations to be closed relative to the vector components, it is necessary to put limitations on two of nine components of vectors $\overline{r_1}$, $\overline{r_2}$, $\overline{r_8}$.

Determining the position of vector $\overline{r_2}$

For determinacy, we assume that parameter $r_{2,z} = \text{const}$ for the component $\overline{r_2}$, and this parameter would be considered as known beforehand (it may be determined based on the known geometry of the surface, enveloping the termini of the vectors in space). From Eq. (12), we get

$$r_{2,x}^2 + r_{2,y}^2 = a_7 , \quad a_7 = \left|\overline{r_2}\right|^2 - r_{2,z}^2 . \tag{14}$$

Taking into account Eq. (8),

$$r_{3,x}\, r_{2,x} + r_{3,y}\, r_{2,y} = a_4 . \tag{15}$$

From Eq. (15), we find

$$r_{2,y} = \left(a_4 - r_{3,x}\ r_{2,x}\right) / r_{3,y} . \tag{16}$$

After substitution in Eq. (14),

$$\left(r_{3,y}^2 + r_{3,x}^2\right) r_{2,x}^2 - 2a_4\ r_{3,x}\ r_{2,x} + a_4^2 - a_7\ r_{3,y}^2 = 0 .$$

Since the vector $\overline{r_3}$ stays on the O_{xy} plane, after substitution $r_{3,x}^2 + r_{3,y}^2 = \left|\overline{r_3}\right|^2$, the last equation looks as follows

$$\left|\overline{r_3}\right|^2\ r_{2,x}^2 - 2a_4\ r_{3,x}\ r_{2,x} + a_4^2 - a_7\ r_{3,y}^2 = 0 . \tag{17}$$

Thus, knowing $r_{2,z}$ and solving Eq. (17), we find $r_{2,x}$ and, by substitution in Eq. (16), we get $r_{2,y}$. Thus, vector $\overline{r_2}$ is $\overline{r_2} = \left\{r_{2,x},\ r_{2,y},\ r_{2,z}\right\}$.

Determining the components of vector $\overline{r_1}$

Equations (9)–(11) are connected to the components of vector $\overline{r_1}$.

Let us introduce the second assumption $r_{1,z} = \text{const}$. From Eq. (11),

$$r_{1,x}^2 + r_{1,y}^2 = a_8 ,\ a_8 = \left|\overline{r_1}\right|^2 - r_{1,z}^2 ,$$

and from Eq. (9),

$$r_{1,x}\ r_{2,x} + r_{1,y}\ r_{2,y} = a_9\ ,\ a_9 = a_5 - r_{1,z}\ r_{2,z} . \tag{18}$$

We get a system of equations for $r_{1,x}$ and $r_{1,y}$

$$\begin{cases} r_{1,x}^2 + r_{1,y}^2 = a_8; \\ r_{1,x}\ r_{2,x} + r_{1,y}\ r_{2,y} = a_9. \end{cases} \tag{19}$$

From Eq. (19), $r_{1,y} = \left(a_9 - r_{1,x}\ r_{2,x}\right) / r_{2,y}$, after substitution in the first equation of system (19), we get

$$r_{1,x}^2 + \left(a_9 - r_{2,x}\ r_{1,x}\right)^2 / r_{2,y}^2 = a_8$$

or

$$r_{2,y}^2\ r_{1,x}^2 + \left(a_9 - r_{2,x}\ r_{1,x}\right)^2 = a_8\ r_{2,y}^2,$$

then

$$\left(r_{2,y}^2 + r_{2,x}^2\right) r_{1,x}^2 - 2a_9\ r_{2,x}\ r_{1,x} + a_9^2 - a_8\ r_{2,y}^2 = 0. \tag{20}$$

Solving quadratic equation (20) for $r_{1,x}$, we get two roots $r_{1,x}$. Substituting one of the roots $r_{1,x}$ into the second equation (19), we get component $r_{1,y}$. Vector $\overline{r}_1 = \left\{ r_{1,x}, r_{1,y}, r_{1,z} \right\}$ is determined.

Determining the components of vector $\overline{r}_8$

Note that components of vector $\overline{r}_8$ are a part of Eqs. (7), (10), and (13).

Thus, we have three equations and three components of vector $\overline{r}_8$, squared components in Eq. (13) make solution more complicated. From Eq. (7), we find $r_{8,y} = \left(a_3 - r_{7,x}\ r_{8,x}\right) / r_{7,y}$, then square this equation: $r_{8,y}^2 = \left(a_3^2 - 2a_3\ r_{7,x}\ r_{8,x} + r_{7,x}^2 + r_{8,x}^2\right) / r_{7,y}^2$. Then $r_{8,y}$ is substituted in Eq. (10) $r_{1,x}\ r_{8,x} + r_{1,y}\left(a_3 - r_{7,x}\ r_{8,x}\right)/r_{7,y} + r_{1,z}\ r_{8,z} = a_6$. After transformation, $\left(r_{1,x}\ r_{7,y} - r_{1,y}\ r_{7,x}\right) r_{8,x} + r_{1,z}\ r_{7,y}\ r_{8,x} = a_6 r_{7,y} - r_{1,y} a_3$.

Then $r_{8,y}^2$ is substituted in Eq. (13)

$$r_{8,y}^2 + \frac{a_3^2 - 2a_3\ r_{7,x}\ r_{8,x} + r_{7,x}^2\ r_{8,x}^2}{r_{7,y}^2} + r_{8,z}^2 = \left|\overline{r}_8\right|^2.$$

This equation becomes as follows

$$\left(r_{7,y}^2 + r_{7,x}^2\right) r_{8,x}^2 - 2a_3\ r_{7,x}\ r_{8,x} + r_{7,y}^2\ r_{8,z}^2 = r_{7,y}^2 \left|r_8\right|^2 - a_3^2.$$

Taking into account $r_{7,y}^2 + r_{7,x}^2 = \left|\overline{r}_7\right|^2$, we get a set of two equations for $r_{8,x}$ and $r_{8,z}$:

$$\begin{cases} \left|\overline{r}_7\right|^2 r_{8,x}^2 - 2a_3\ r_{7,x}\ r_{8,x} + r_{7,y}^2\ r_{8,z}^2 = r_{7,y}^2 \left|\overline{r}_8\right|^2 - a_3^2; \\ \left(r_{1,x}\ r_{7,y} - r_{1,y}\ r_{7,x}\right) r_{8,x} + r_{1,z}\ r_{7,y}\ r_{8,z} = a_6\ r_{7,y} - r_{1,y}\ a_3. \end{cases} \tag{21}$$

From the second equation of system (21), we define the product $r_{7,y}\ r_{8,z}$:

$$r_{7,y}\ r_{8,z} = \left[\left(a_6\ r_{7,y} - r_{1,y}\ a_3\right) - \left(r_{1,x}\ r_{7,y} - r_{1,y}\ r_{7,x}\right) r_{8,x}\right] / r_{1,z}.$$

If $\gamma_1 = \left(a_6\ r_{7,y} - r_{1,y}\ a_3\right)$, $\gamma_2 = \left(r_{1,x}\ r_{7,y} - r_{1,y}\ r_{7,x}\right)$, then $r_{7,y}\ r_{8,z} = \left(\gamma_1 - \gamma_2\ r_{8,x}\right) / r_{1,z}$. We square $r_{7,y}\ r_{8,z}$:

$$\left(r_{7,y}\ r_{8,z}\right)^2 = \left(\gamma_1^2 - 2\gamma_1\ \gamma_2\ r_{8,x} + \gamma_2^2\ r_{8,x}^2\right) / r_{1,z}^2. \tag{22}$$

Substituting Eq. (22) in the first equation of system (21), we get

$$\left|\overline{r}_7\right|^2 r_{8,x}^2 - 2a_3\ r_{7,x}\, r_{8,x} + \frac{\gamma_1^2 - 2\gamma_1\ \gamma_2\ r_{8,x} + \gamma_2^2\ r_{8,x}^2}{r_{1,z}^2} = r_{7,y}^2 \left|\overline{r}_8\right|^2 - a_3^2\,,$$

$$r_{1,z}^2 \left|\overline{r}_7\right|^2 r_{8,x}^2 - 2a_3\ r_{7,x}\, r_{1,z}^2\ r_{8,x} + \gamma_1^2 - 2\gamma_1\,\gamma_2\ r_{8,x} + \gamma_2^2\ r_{8,x}^2 + a_3^2\ r_{1,z}^2 - r_{1,z}^2\ r_{7,y}^2 \left|\overline{r}_8\right|^2 = 0\,.$$

As a result, we get a quadratic equation for $r_{8,x}$

$$\left(r_{1,z}^2 \left|\overline{r}_7\right|^2 + \gamma_2^2\right) r_{8,x}^2 - 2\left(\gamma_1\,\gamma_2 + a_3\ r_{7,x}\, r_{1,z}^2\right) r_{8,x} + \gamma_1^2 + r_{1,z}^2 \left(a_3^2 - r_{7,y}^2 \left|\overline{r}_8\right|^2\right) = 0\,, \tag{23}$$

and solving this equation, we get two roots for component $r_{8,x}$.

From Eq. (7), we get

$$r_{8,y} = \left(a_3 - r_{7,x}\ r_{8,x}\right) \big/ r_{7,y} \tag{24}$$

and, from Eq. (13),

$$r_{8,z} = \left(\left|\overline{r}_8\right|^2 - r_{8,x}^2 - r_{8,y}^2\right)^{1/2}. \tag{25}$$

Thus, vector $\overline{r}_8$ is as follows: $\overline{r}_8 = \left\{ r_{8,x},\, r_{8,y},\, r_{8,z} \right\}$.

EXAMPLE OF SOLUTION

The initial data is as follows: $\left|\overline{r}_5\right| = 8.8$; $\left|\overline{r}_3\right| = \left|\overline{r}_7\right| = 6.6$; $\left|\overline{r}_2\right| = \left|\overline{r}_8\right| = 13.2$; $\left|\overline{r}_1\right| = 8.8$.

The areas of triangles on the plane are determined by these data:

$$S_{53}^0 = S_{57}^0 = S_{87}^0 = S_{32}^0 = S_{87}^0 = S_{32}^0 = 26.4.$$

Let us assume $\overline{r}_3 = \{-5;\, -4.31;\, 0\}$, $\overline{r}_7 = \{-4.31;\, -5;\, 0\}$.

For Eqs. (1) and (2),

$$a_1 = a_2 = \sqrt{\left|\overline{r}_5\right|^2\ \left|\overline{r}_3\right|^2 - 4\left(S_{53}^0\right)^2} = \sqrt{8.8^2 \times 6.6^2 - 4 \times 26.4^2} = 24.196\,.$$

Components of vector $\overline{r}_5$ may be found using Eqs. (3) and (4)

$$r_{5,x} = \frac{-a_1\, r_{7,y} + a_2\, r_{3,y}}{r_{3,x}\ r_{7,y} - r_{3,y}\ r_{7,x}} = 0.383128\,;\ \ r_{5,y} = \frac{-a_2\, r_{3,x} + a_1\, r_{7,x}}{r_{3,x}\ r_{7,y} - r_{3,y}\ r_{7,x}} = 5.169457\,;$$

$$r_{5,z} = \sqrt{\left|\overline{r}_5\right|^2 - r_{5,x}^2 - r_{5,y}^2} = 7.111254\,;\ \ \overline{r}_5 = \{0.383;\ 5.169;\ 7.111\}\,.$$

Considering vector $\overline{r}_2$, let's assume $r_{2,z} = 5$.

For Eqs. (7) and (8), we find

$$a_3 = a_4 = \sqrt{\left|\overline{r}_7\right|^2\ \left|\overline{r}_8\right|^2 - 4\left(S_{78}^0\right)^2} = \sqrt{6.6^2 \times 13.2^2 - 4 \times 26.4^2} = 60.297\,,$$

$$a_5 = a_6 = \sqrt{\left|\overline{r}_2\right|^2\ \left|\overline{r}_1\right|^2 - 4\left(S_{21}^0\right)^2} = \sqrt{8.8^2 \times 13.2^2 - 4 \times 26.4^2} = 103.470\,.$$

From the solution of Eq. (17)

$$\left|\overline{r}_3\right|^2\ r_{2,x}^2 - 2a_4\ r_{3,x}\ r_{2,x} + a_4^2 - a_7\ r_{3,y}^2 = 0\,,$$

where $a_7 = |\overline{r_2}|^2 - r_{2,z}^2$, we find $r_{2,x} = -3.878$ and, substituting in Eq. (16), we get

$$r_{2,y} = \left(a_4 - r_{3,x}\ r_{2,x}\right) / r_{3,y} = -11.585 .$$

Thus, vector $\overline{r_2}$ is defined:

$$\overline{r_2} = \left\{ r_{2,x},\ r_{2,y},\ r_{2,z} \right\} = \left\{ -3.878, \quad -11.585, \quad 5 \right\} .$$

Proceeding to vector $\overline{r_1}$, we assume $r_{1,z} = 0.5$.

Solve quadratic equation (20) for $r_{1,x}$:

$$\left(r_{2,y}^2 + r_{2,x}^2\right) r_{1,x}^2 - 2a_9\ r_{2,x}\ r_{1,x} + a_9^2 - a_8\ r_{2,y}^2 = 0 ,$$

where $a_8 = |\overline{r_1}|^2 - r_{1,z}^2 = 77.19$ (Eq. (18)), $a_9 = a_5 - r_{1,z}\ r_{2,z} = 100.97$. The root is $r_{1,x} = 0.203$.

From Eq. (19), we find

$$r_{1,y} = \left(a_9 - r_{1,x}\ r_{2,x}\right) / r_{2,y} = -8.78 .$$

Vector $\overline{r_1} = \left\{ r_{1,x},\ r_{1,y},\ r_{1,z} \right\} = \left\{ 0.203, -8.78, 0.5 \right\}$ is determined.

Let us define components for vector $\overline{r_8}$. For Eq. (22), we define parameters

$$\gamma_1 = a_6\ r_{7,y} - r_{1,y}\ a_3 = 91.33 ,$$

$$\gamma_2 = r_{1,x}\ r_{7,y} - r_{1,y}\ r_{7,x} = 36.83 .$$

We find expressions in the brackets in Eq.(23):

—the first expression is

$$\left(r_{1,z}^2 |\overline{r_7}|^2 + \gamma_2^2\right) = 1367.08 ;$$

—the second expression is

$$-2\left(\gamma_1\ \gamma_2 + a_3\, r_{7,x}\ r_{1,z}^2\right) = -6876.21 ;$$

—the third expression is

$$\gamma_1^2 + r_{1,z}^2 \left(a_3^2 - r_{7,y}^2 |\overline{r_8}|^2\right) = 6052.18 .$$

Then Eq. (23) in numeric expression looks as follows:

$$1367.08\ r_{8,x}^2 - 6876.21\ r_{8,x} + 6052.18 = 0 .$$

Solving this equation, we obtain the root $r_{8,x} = 3.89$ and from Eq. (24) $r_{8,y} = \left(a_3 - r_{7,x}\ r_{8,x}\right) / r_{7,y} = -10.51$ and $r_{8,z} = \left(|\overline{r_8}|^2 - r_{8,x}^2 - r_{8,y}^2\right)^{1/2} = 6.98$. Thus, $\overline{r_8} = \left\{ r_{8,x},\ r_{8,y}\ \ r_{8,z} \right\} = \left\{ 3.89, -10.51,\ 6.98 \right\}$.

Vectors $\overline{r_5}$, $\overline{r_2}$, $\overline{r_1}$, and $\overline{r_8}$ in the space O_{xyz} are defined.

In general, this vector model provides a possibility to define the relief of the structure at any stage of transformation as well as the shape of the enveloping surface. Figure 3 shows the stages of transformation of the hexactinal structure with aforementioned parameters.

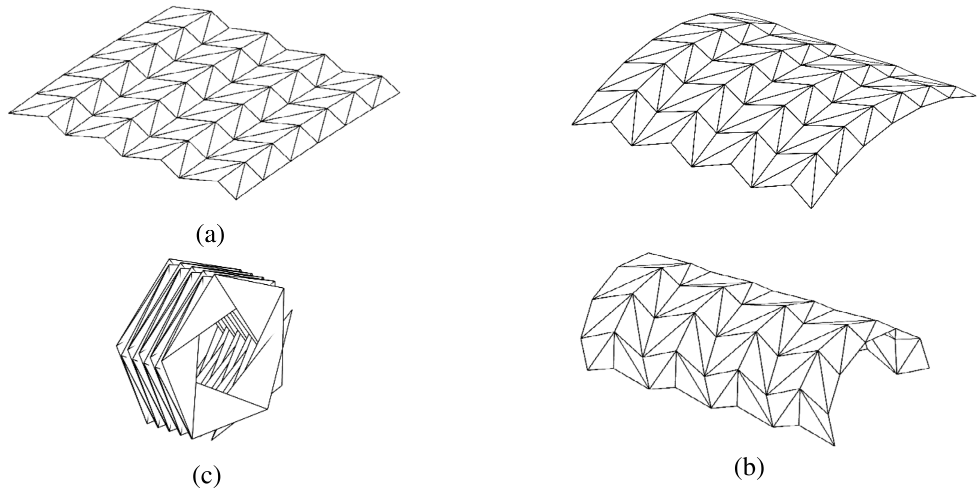

Fig. 3. Transformation of a hexactinal structure: (a)—the developed view; (b)—the stages of transformation; (c)—the finite compact structure.

CONCLUSIONS

A vector model for transformation of the hexactinal structure is presented. It provides a possibility to define the geometry of the relief of the structure at any stage of transformation as well as define the motion kinematics of all its elements. The example of calculation of the fixed position of the folded structure elementary unit and numerical simulation confirm efficiency of the developed algorithms. The technique suggested may become the basis for constructing the models of other hexactinal structures, including modified ones.

Results of the research may be used to design the cores of sandwich shells, sound absorbing structures and development of the parts of equipment for manufacturing of such structures.

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